Solutions Hwk 2

1.4)

34) Consider A to be $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ such that $AB = BA$ for any $B_{2\times 2}$

Then
$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} a & 0 \\ c & 0 \end{bmatrix} = \begin{bmatrix} a & b \\ 0 & 0 \end{bmatrix}$$

$\therefore$ $b = c = 0$, $A = \begin{bmatrix} a & 0 \\ 0 & d \end{bmatrix}$

Also
$$\begin{bmatrix} a & 0 \\ 0 & d \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} a & 0 \\ 0 & d \end{bmatrix}$$

$$\begin{bmatrix} a & a \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} a & d \\ 0 & 0 \end{bmatrix}$$

$\Rightarrow a = d$ $\therefore$ $A = \begin{bmatrix} a & 0 \\ 0 & a \end{bmatrix}$ or $= A = a\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

or $A = aI$ where a is any scalar and I is the identity matrix

1.5)

11) Consider $P = 0$

$$(A^T)^0 = I_n = (I_n)^T = (A^0)^T$$

$\therefore$ True for $P = 0$

now consider it to be true for $P = n$

Then $(A^T)^{n+1} = (A^T)^n \cdot A^T$

$$= (A^n)^T \cdot A^T$$

$$= (A A^n)^T \quad \{(AB)^T = B^T A^T\}$$

$$= (A^{n+1})^T$$

hence $(A^T)^P = (A^P)^T$

33a)

$$A = \begin{bmatrix} 1 & 3 \\ 5 & 2 \end{bmatrix}$$

determinant of $A = 2 - 15 = -13$

$$\text{adj}[A] = \begin{bmatrix} 2 & -3 \\ -5 & 1 \end{bmatrix}$$

{interchange the elements on the main diagonal & change the sign of elements on the other diagonal. works only for 2×2 matrices}

$$A^{-1} = \frac{1}{|A|} \text{adj} A = \begin{bmatrix} -2/13 & 3/13 \\ 5/13 & -1/13 \end{bmatrix}$$

53) when $AX = 0$

$\Rightarrow A^{-1}(AX) = A^{-1}0 = 0.$ $\left[\underbrace{A^{-1}(AX) = (A^{-1}A)X}_{\text{by associativity of Matrices}} = IX \text{ \& hence } X = 0\right]$

This is possible only when $X = 0$

②

1.6).10). $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}$ $f(x) = Ax$

$w = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

it is equivalent to ask whether there is a vector $v = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$ such that $f(v) = w$

now $v_1 + 2v_2 = 1$
$0v_1 + v_2 = 1$
$v_1 + v_2 = 1$

There exists no non trivial solution for these system of equations which implies w is NOT in the range of f.

19) a) T is a counterclockwise rotation by 60 degrees

b) T is clockwise rotation by 30 degrees

c) the smallest possible $k = 360/30 = \underline{\underline{12}}$

2.1)

8) a) REF
b) RREF
c) N

$\Bigg\}$ by the rules (a), (b), (c), (d) given in the text