Math 340
Lecture 2 G.Meyer

Exam 3

1. (24 points; 3 points each) Answer the following short answer questions (no justification wanted). If the information given is not enough to uniquely determine the answer, write undetermined. Let A be an n by n matrix with eigenvalues (including multiplicities) -1, -1, 4, 4, 4.

    1. What is n?
    2. The determinant of A is:
    3. The coefficient of λ^4 in the characteristic polynomial of A is:
    4. The dimension of the row space of A is:
    5. The eigenvalues of the matrix A^2 of A are:
    6. Is A invertible?
    7. The dimension of the eigenspace of A for the eigenvalue -1 is:
    8. Is A diagonalizable?

2. (6+12+8 =26 points) Consider the homogeneous system of 2 equation in 4 unknowns with coefficient matrix

$$A = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \end{bmatrix}$$

    1. Determine a basis for the null space U of A.

    2. Use the Gram-Schmidt process to determine a orthonormal basis of U (Show your work!)

    3. Show that the vector $w = \begin{bmatrix} -2 \\ 2 \\ -1 \\ 3 \end{bmatrix}$ is in U and write it as a linear combination of the orthonormal basis found above.

3. (10+10+10=30 points) Let A, B and C be real, nxn matrices. Prove the following three simple assertions:

   1. If A is similar to B, and B is similar to C, then A is similar to C.

   2. If 2 is an eigenvalue of A, then 4 is an eigenvalue of

   $$2A^3 - 5A^2 + 4A.$$

   3. If A has n linearly independent eigenvectors $u_1, u_2, ..., u_n$, then A is diagonalizable.

4. (12 points; 3 points each) Let A be a 6 by 9 matrix of rank 4. Answer the following questions:

   1. The dimension of the column space of A is:
   2. The dimension of the row space of A is:
   3. The dimension of the null space of A is:
   4. The dimension of the null space of A^T is:

5. (6+8+12=26 points) Consider the linear transformation $L : R^3 \to R^3$ given by:

   $$L\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} x_1 + 2x_2 - x_3 \\ x_1 - 2x_2 + 2x_3 \\ x_1 + x_2 + x_3 \end{bmatrix}$$

   .

   1. What is the matrix of L with respect to the standard basis $S : e_1, e_2, e_3$ of R^3?

   2. Consider the basis $T : u_1 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} u_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, u_3 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix},.$ What is the transition matrix from T to S?

   3. What is the matrix of L with respect to the basis T?

6. (20 points) Determine a least squares solution to

   $$\begin{bmatrix} 1 & 1 \\ 2 & -1 \\ 1 & 2 \\ -1 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}.$$

7. Extra Credit: (another 5 points to the sum of all your course scores) Circle whether the following assertions are True or False:

   1. T F A real, square matrix always has at least one real eigenvalue.

   2. T F A finite dimensional vector space with an inner product always has an orthonormal basis.

   3. T F Every real, symmetric matrix is diagonalizable.

   4. T F If P is an orthogonal matrix, then $|detP| = 1$.

   5. T F If u is orthogonal to vectors v and w, then u is orthogonal to every linear combination of v and w.