Solutions to HWK 4

3.1)

12b)

$$A = \begin{bmatrix} 2 & 4 & 5 \\ 0 & -6 & 2 \\ 0 & 0 & 3 \end{bmatrix}$$

$\det A = 0 + 0 + 3\,(2\times -6 - 4\times 0)$

$\boxed{\det A = -36.}$

14) a) $\begin{vmatrix} t & 4 \\ 5 & t-8 \end{vmatrix}$

$\det = 0 \Rightarrow t(t-8) - 4\times 5 = 0$

$\boxed{t^2 - 8t - 20 = 0}$ or $t = -2$ or $t = 10$

b) $\det \begin{bmatrix} t-1 & 0 & 1 \\ -2 & t & -1 \\ 0 & 0 & t+1 \end{bmatrix} = 0$

$\Rightarrow t-1\,(t(t+1))$ ~~$\cancel{t}$~~

$\Rightarrow t(t^2-1)$

$\Rightarrow \boxed{t^3 - t = 0}$ $\quad t = 0$ or 1 or -1

3.2) 4)

$$A = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = -2$$

we know that

$$\det(A_{kr_i + r_j \to r_j}) = \det A$$

In this case its true as

$$\begin{vmatrix} a_1 - 1/2\, a_3 & a_2 & a_3 \\ b_1 - 1/2\, b_3 & b_2 & b_3 \\ c_1 - 1/2\, c_3 & c_2 & c_3 \end{vmatrix}$$ is got by $R_1 - 1/2 R_3$ of A which is R_1

hence

$$\begin{vmatrix} a_1 - 1/2\, a_3 & a_2 & a_3 \\ b_1 - 1/2\, b_3 & b_2 & b_3 \\ c_1 - 1/2\, c_3 & c_2 & c_3 \end{vmatrix} = |A| = -2$$

(5)

a) By Corollary 3.3

$\det(A^{-1}) = 1/\det(A)$. Since $A = A^{-1}$, we have

$\det(A) = 1/\det(A) \Rightarrow (\det(A))^2 = 1 \Rightarrow \boxed{\det(A) = \pm 1}$

b) if $A^T = A^{-1}$, then $\det(A^T) = \det(A^{-1})$ but

$\det(A) = \det(A^T)$ and $\det(A^{-1}) = 1/\det(A)$

hence we have $\det(A) = \dfrac{1}{\det(A)} \Rightarrow (\det(A))^2 = 1 \Rightarrow \boxed{\det(A) = \pm 1}$

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32) If $A^2 = A$, then $\det(A^2) = \det(A)$,

So $[\det(A)]^2 = \det(A)$. Thus $\det(A)\cdot(\det(A)-1) = 0 \Rightarrow$

$\det(A) = 0$ or $\det(A) = 1$

3.3)

6) 3.21b) $\begin{vmatrix} 2 & 1 \\ 4 & 3 \end{vmatrix}$

$= 2{\times}3 - 1{\times}4 = 2$

1c) $\begin{vmatrix} 4 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{vmatrix} = 4\begin{vmatrix} 2 & 0 \\ 0 & 3 \end{vmatrix} + 0 + 0$

$= 24$

f) $\begin{vmatrix} 4 & 2 & 3 & -4 \\ 3 & -2 & 1 & 5 \\ -2 & 0 & 1 & -3 \\ 8 & -2 & 6 & 4 \end{vmatrix}$ $R_1 + R_2 \rightarrow R_2$

$R_1 + R_4 \rightarrow R_4$

$\begin{vmatrix} 4 & 2 & 3 & -4 \\ 7 & 0 & 4 & 1 \\ -2 & 0 & 1 & -3 \\ 12 & 0 & 9 & 0 \end{vmatrix} = 4\,[0] + (-2)\begin{vmatrix} 7 & 4 & 1 \\ -2 & 1 & -3 \\ 12 & 9 & 0 \end{vmatrix}$

$+3\,[0] + (-4)\,[0]$

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$$= -2\begin{bmatrix} 7 & 4 & 1 \\ -2 & 1 & -3 \\ 12 & 9 & 0 \end{bmatrix}$$

$$= -2\left[1\begin{pmatrix} -2 & 1 \\ 12 & 9 \end{pmatrix} + 3\begin{pmatrix} 7 & 4 \\ 12 & 9 \end{pmatrix} + 0\right]$$

$$= \underline{\underline{-30}}$$

13) a) from 3.2, each term in expansion of det of $n \times n$ is a product of n entries. Each of these products contains exactly one entry from each row and exactly one entry from each column. Thus each product from $\det(tI_n - A)$ contains at most n terms of the form $t - a_{ii}$. Hence each of these is at most a polynomial of degree n. Since one of products has the form $(t-a_{11})(t-a_{22})\cdots(t-a_{nn})$ it follows that sum of the products is a polynomial of degree n in t.

b) Coeff of t^n is $\boxed{1}$ as it appears only in the term $(t-a_{11})(t-a_{22})\cdots(t-a_{nn})$.

c) using part (a)

$\det(tI_n - A) = t^n + c_1 t^{n-1} + \cdots c_{n-1}t + c_n$

set $t=0 \Rightarrow \det(-A) = c_n$ $\therefore$ $\boxed{c_n = (-1)^n \det(A)}$