§2.2.

3(a). $\begin{cases} x = 2-s, \\ y = s \\ z = -3-t \\ w = t \end{cases}$

8(b). $\begin{cases} x = 1-r \\ y = 2+r \\ z = -1+r \\ x_4 = r, \text{ any real number} \end{cases}$

10. $\vec{x} = \begin{pmatrix} r \\ 0 \end{pmatrix}$ where $r \neq 0$.

15. (a). $a = \pm\sqrt{3}$

(b). $a \neq \pm\sqrt{3}$

(c). There is no value of a. st. the system has infinitly many solns.

19. ($\Rightarrow$) If $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is row equivalent to I_2, then a & c can't be zero at the same time;

Without loss of generality, we assume $a \neq 0$.

Then we can perform row operations.

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \to \begin{pmatrix} 1 & \frac{b}{a} \\ c & d \end{pmatrix} \to \begin{pmatrix} 1 & \frac{b}{a} \\ 0 & d-\frac{bc}{a} \end{pmatrix}$$

Again $A \sim I_2 \Rightarrow d - \frac{bc}{a} \neq 0 \Rightarrow ad-bc \neq 0$.

19 ($\Leftarrow$) Conversely, if $ad-bc \neq 0$,

then a, c can't be zero at the same time

(otherwise $0\cdot d - b\cdot 0 = 0$, not allowed).

Without loss of generality, we can assume $a \neq 0$

Then we can again perform row operations.

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \to \begin{pmatrix} 1 & \frac{b}{a} \\ c & d \end{pmatrix} \to \begin{pmatrix} 1 & \frac{b}{a} \\ 0 & d-\frac{bc}{a} \end{pmatrix}$$

Now, since $ad-bc \neq 0 \Rightarrow d - \frac{bc}{a} \neq 0$

we can divide the 2nd row by $(d-\frac{bc}{a})$, and get

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \to \begin{pmatrix} 1 & \frac{b}{a} \\ 0 & 1 \end{pmatrix} \to \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Therefore $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is row equivalent to I_2. #.

22. $-a+b+c = 0$

29. (b). Let x_p be any particular soln to $Ax=b$

x be any other soln to $Ax=b$.

Then let $x_h = x - x_p$,

Then $Ax_h = A(x-x_p) = Ax - Ax_p = 0$

and $x = (x-x_p) + x_p = x_h + x_p$ #.

42. No soln.

§2.3.

4(a). $A \sim \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = C$

(b). $B = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{pmatrix}$

(c) $A \cdot B = I_3$

$B \cdot A = I_3$

7. $A^{-1} = \begin{pmatrix} -2 & \frac{3}{2} \\ 1 & -\frac{1}{2} \end{pmatrix}$

9. (a) singular

(b). $A^{-1} = \begin{pmatrix} \frac{1}{2} & -\frac{1}{4} \\ \frac{1}{6} & \frac{1}{12} \end{pmatrix}$

(c) $A^{-1} = \begin{pmatrix} 0 & 1 & -1 \\ 2 & -2 & -1 \\ -1 & 1 & 1 \end{pmatrix}$

(d) singular.

10. (a) singular

(b). $\begin{pmatrix} 1 & -1 & 0 \\ 1 & -2 & 1 \\ -\frac{3}{2} & \frac{5}{2} & -\frac{1}{2} \end{pmatrix}$

(c). $\begin{pmatrix} -1 & \frac{3}{2} & \frac{1}{2} \\ 1 & -\frac{3}{2} & \frac{1}{2} \\ 0 & \frac{1}{2} & -\frac{1}{2} \end{pmatrix}$

10. (d). $\begin{pmatrix} \frac{3}{5} & -\frac{3}{5} & -\frac{1}{5} \\ \frac{2}{5} & \frac{3}{5} & -\frac{4}{5} \\ -\frac{1}{5} & \frac{1}{5} & \frac{2}{5} \end{pmatrix}$

16. $A = \begin{pmatrix} \frac{3}{2} & -1 & \frac{1}{2} \\ \frac{1}{2} & 0 & -\frac{1}{2} \\ -1 & 1 & 0 \end{pmatrix}$

17. (a) & (b)

23. pf. The matrices A and B are row equivalent if and only if $B = E_k \cdot E_{k-1} \cdots E_2 \cdot E_1 \cdot A$. Then let $P = E_k E_{k-1} \cdots E_2 E_1$, which is nonsingular (Thm 2.8).

28. If A has a row of zeros, then A can't be row equivalent to I_n, so by corollary 2.2, A is singular.

If jth column of A is the zero column, then the homogeneous system $A \cdot \vec{x} = 0$ has nontrivial soln, the vector $\vec{x}$ with 1 in the j-th entry and zeros elsewhere. By Thm 2.9, A is singular. #

§2.4.

2(a). $\begin{pmatrix} I_4 \\ O_{1\times 4} \end{pmatrix}$

6. $B = \begin{pmatrix} I_2 & 0 \\ 0 & 0 \end{pmatrix}$

and $B = \begin{pmatrix} -1 & 2 & 0 \\ 1 & -1 & 0 \\ -1 & 1 & 1 \end{pmatrix} \cdot A \cdot \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}$

8. ($\Rightarrow$) If A is equivalent to $O_{m\times n}$

Then $A = P_{m\times m} \cdot O \cdot Q_{n\times n}$ for nonsingular matrices P & Q. (Thm 2.13).

But $P \cdot O_{m\times n} \cdot Q = O_{m\times n}$, so

$A = O$.

($\Leftarrow$) conversely, if $A = O$, then

A is of course equivalent to O,

which is the definition (Def 2.5).

10. possible answer.

(a) $\begin{pmatrix} 1 & -2 & 3 & 0 \\ 0 & -1 & 4 & 3 \\ 0 & 2 & -5 & 2 \end{pmatrix}$