Solution to HW 1.

§1.2.

4. $a=3,\ b=1,\ c=8,\ d=-2.$

6. (c). $\begin{bmatrix} 7 & -7 \\ 0 & 1 \end{bmatrix}$, (f) Impossible.

9. (a) $\begin{bmatrix} 2 & 4 \\ 4 & 2 \\ 6 & 8 \end{bmatrix}$. (b) Impossible.

12. $\begin{bmatrix} \lambda-1 & -2 & -3 \\ -6 & \lambda+2 & -3 \\ -5 & -2 & \lambda-4 \end{bmatrix}$

15. The entries are symmetric about the main diagonal.

17. (a). $\sum_{i=1}^{n} (r_i + s_i)\cdot a_i$

$= \sum_{i=1}^{n} (r_i \cdot a_i + s_i \cdot a_i)$

$= \sum_{i=1}^{n} r_i \cdot a_i + \sum_{i=1}^{n} s_i \cdot a_i$

(b). $\sum_{i=1}^{n} c\cdot(r_i a_i)$

$= c\cdot r_1 a_1 + c\cdot r_2 a_2 + \cdots + c\cdot r_n a_n$

$= c\cdot(r_1 a_1 + r_2 a_2 + \cdots + r_n a_n)$

$= c\cdot \sum_{i=1}^{n} r_i a_i$

19. (a). True. $\sum_{i=1}^{n}(a_i+1) = \sum_{i=1}^{n} a_i + \sum_{i=1}^{n} 1$

$= \sum_{i=1}^{n} a_i + n.$

(b) True. $\sum_{i=1}^{n}\left(\sum_{j=1}^{m} 1\right) = \sum_{i=1}^{n} m = m\cdot n$

(c) True. $\sum_{j=1}^{m}\left(\sum_{i=1}^{n} a_i b_j\right)$

$= \sum_{j=1}^{m}\left(b_j \sum_{i=1}^{n} a_i\right)$

$= b_1\cdot\left(\sum_{i=1}^{n} a_i\right) + b_2\left(\sum_{i=1}^{n} a_i\right) + \cdots + b_m\left(\sum_{i=1}^{n} a_i\right)$

$= (b_1 + b_2 + \cdots + b_m)\left(\sum_{i=1}^{n} a_i\right)$

$= \left(\sum_{j=1}^{m} b_j\right)\cdot\left(\sum_{i=1}^{n} a_i\right) = \left(\sum_{i=1}^{n} a_i\right)\cdot\left(\sum_{j=1}^{m} b_j\right).$

§1.3.

1. (c) 4

5. $x=4,\ y=-6$

8. $x=\pm 5.$

14. (a) $\begin{bmatrix} 58 & 12 \\ 66 & 13 \end{bmatrix}$, (c) $\begin{bmatrix} 28 & 8 & 38 \\ 34 & 4 & 41 \end{bmatrix}$

(f). $\begin{bmatrix} -4 & -8 & -26 \\ -30 & 0 & -31 \end{bmatrix}$

19. $A\cdot B = \begin{bmatrix} -4 & 7 \\ 0 & 5 \end{bmatrix}$, $B\cdot A = \begin{bmatrix} -1 & 2 \\ 9 & 2 \end{bmatrix}.$

24. $\mathrm{col}_1(A\cdot B) = 1\cdot\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + 3\cdot\begin{bmatrix} -2 \\ 4 \\ 0 \end{bmatrix} + 2\begin{bmatrix} -1 \\ 3 \\ -2 \end{bmatrix},$

$\mathrm{col}_2(AB) = -1\cdot\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + 2\begin{bmatrix} -2 \\ 4 \\ 0 \end{bmatrix} + 4\begin{bmatrix} -1 \\ 3 \\ -2 \end{bmatrix}.$

30. (a). $\begin{bmatrix} 2 & 3 & -3 & 1 & 1 \\ 3 & 0 & 2 & 0 & 3 \\ 2 & 3 & 0 & -4 & 0 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} 2 & 3 & -3 & 1 & 1 \\ 3 & 0 & 2 & 0 & 3 \\ 2 & 3 & 0 & -4 & 0 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix}\cdot\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 7 \\ -2 \\ 3 \\ 5 \end{bmatrix}$

(c). $\left[\begin{array}{ccccc|c} 2 & 3 & -3 & 1 & 1 & 7 \\ 3 & 0 & 2 & 0 & 3 & -2 \\ 2 & 3 & 0 & -4 & 0 & 3 \\ 0 & 0 & 1 & 1 & 1 & 5 \end{array}\right]$

37. (a). $x = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ is the only solution.

43. (a). $\mathrm{Tr}(c\cdot A) = \sum_{i=1}^{n}(c\cdot a_{ii}) = c\cdot\sum_{i=1}^{n} a_{ii} = c\cdot \mathrm{Tr}(A).$

(b). $\mathrm{Tr}(A+B) = \sum_{i=1}^{n}(a_{ii}+b_{ii}) = \sum_{i=1}^{n} a_{ii} + \sum_{i=1}^{n} b_{ii}$

$= \mathrm{Tr}(A) + \mathrm{Tr}(B).$

(c). Let $A\cdot B = C = [c_{ij}]$. Then

$\mathrm{Tr}(AB) = \sum_{i=1}^{n} c_{ii} = \sum_{i=1}^{n}\left(\sum_{k=1}^{m} a_{ik}\cdot b_{ki}\right)$

$= \sum_{k=1}^{m}\left(\sum_{i=1}^{n} b_{ki}\cdot a_{ik}\right) = \mathrm{Tr}(B\cdot A)$

§1.3.

43. (d). Since $a^T_{ii} = a_{ii}$,

$$Tr(A^T) = \sum_{i=1}^{n} a^T_{ii} = \sum_{i=1}^{n} a_{ii} = Tr(A)$$

(e). Let $A^T \cdot A = B = [b_{ij}]$. Then

$$b_{ii} = \sum_{j=1}^{n} a^T_{ij} \cdot a_{ji} = \sum_{j=1}^{n} a_{ji}^2 \geq 0$$

$$\Rightarrow Tr(B) = \sum_{i=1}^{n} b_{ii} = \sum_{i=1}^{n} \left(\sum_{j=1}^{n} a_{ji}^2 \right) \geq 0$$

46. (a). Let $A = [a_{ij}]_{m \times p}$, $B = [b_{ij}]_{p \times n}$,

Then $b_j = \begin{bmatrix} b_{1j} \\ b_{2j} \\ \vdots \\ b_{pj} \end{bmatrix}$ and the i-th entry

of Ab_j is $\sum_{k=1}^{p} a_{ik} \cdot b_{kj}$, which is exactly

the (i,j) entry of $A \cdot B$.

(b). The i-th row of AB is

$$\left[\sum_{k} a_{ik} b_{k1} \quad \sum_{k=1} a_{ik} b_{k2} \quad \cdots \quad \sum_{k=1}^{n} a_{ik} \cdot b_{kn} \right].$$

Also. since $a_i = [a_{i1} \quad a_{i2} \quad \cdots \quad a_{ip}]$, we have

$$a_i \cdot B = \left[\sum_{k=1}^{n} a_{ik} b_{k1} \quad \sum_{k=1}^{n} a_{ik} b_{k2} \quad \cdots \quad \sum_{k=1}^{n} a_{ik} b_{kn} \right]$$

which is exactly the i-th row of AB.