§4.6.

(2) (a) (b) (d) are not basis for $\mathbb{R}^3$.

(c) is a basis for $\mathbb{R}^3$.

(4) (b) not not bases for P_2.

(c). is basis for P_2.

(8). (a) is Not a basis of $\mathbb{R}^3$

(b) is a basis of $\mathbb{R}^3$.

$$\begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} + 2\begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix} - \begin{pmatrix} 3 \\ 4 \\ -1 \end{pmatrix}$$

(10). (a). is a basis for P_2.

$$5t^2 - 3t + 8 = -3(t^2+t) + 0\cdot t^2 + 8(t^2+1)$$

(b) is NOT a basis for P_2

(12). A possible basis is

$\{(1,1,0,-1), (0,1,2,1), (1,0,1,-1)\}$.

$\dim W = 3$

(15) $a \neq 0, \pm 1$

(20). (a). $S = \left\{ \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right\}$

(b). $S = \left\{ \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ -1 \\ 1 \end{pmatrix} \right\}$

(c). $S = \left\{ \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -5 \\ 0 \\ 1 \end{pmatrix} \right\}$.

(24). (a). $\dim W = 3$

(b). $\dim W = 2$.

(30) a basis for $M_{2\cdot 3} = \left\{ \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \right\}$.

$\dim(M_{2\cdot 3}) = 2\cdot 3 = 6$.

$\dim M_{m\times n} = m \times n$.

§4.6

39. Let $T = \{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_m\}$, $m > n$ be a set of vectors in V. Since $m > n$, Theorem 4.10 implies that T is linearly dependent.

42. Let $\dim V = \dim W = n$. Let $S = \{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n\}$ be a basis for W. Then S is also a basis for V, by Theorem 4.13. Hence, $V = W$.

45. (a) If span $S \neq V$, then there exists a vector $\mathbf{v}$ in V that is not in S. Vector $\mathbf{v}$ cannot be the zero vector since the zero vector is in every subspace and hence in span S. Hence $S_1 = \{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n, \mathbf{v}\}$ is a linearly independent set. This follows since $\mathbf{v}_i$, $i = 1, \ldots, n$ are linearly independent and $\mathbf{v}$ is not a linear combination of the $\mathbf{v}_i$. But this contradicts Corollary 4.4. Hence our assumption that span $S \neq V$ is incorrect. Thus span $S = V$. Since S is linearly independent and spans V it is a basis for V.

    (b) We want to show that S is linearly independent. Suppose S is linearly dependent. Then there is a subset of S consisting of at most $n - 1$ vectors which is a basis for V. (This follows from Theorem 4.9) But this contradicts $\dim V = n$. Hence our assumption is false and S is linearly independent. Since S spans V and is linearly independent it is a basis for V.

§4.7.

26. Since the reduced row echelon forms of matrices A and B are the same it follows that the solutions to the linear systems $A\mathbf{x} = \mathbf{0}$ and $B\mathbf{x} = \mathbf{0}$ are the same set of vectors. Hence the null spaces of A and B are the same.

§4.7.

(2) (a). solution space $= \left\{ \begin{pmatrix} -r+2s \\ r \\ s \end{pmatrix} = r\begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} + s\begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} \right\}$

(b). $\begin{pmatrix} -r+2s \\ r \\ s \end{pmatrix} = r\begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} + s\begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} = r\cdot\vec{v}_1 + s\cdot\vec{v}_2$.

(6) a basis $= \left\{ \begin{pmatrix} 4 \\ 3 \\ -2 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ -2 \\ 0 \\ 1 \end{pmatrix} \right\}$

dimension = 2.

(12) a basis $= \left\{ \begin{pmatrix} 0 \\ 3 \\ 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ -6 \\ -3 \\ 0 \\ 1 \end{pmatrix} \right\}$

(15). a basis $= \left\{ \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} \right\}$.

(18). $\lambda = 3, -2$

(22). $\vec{x} = \vec{x}_p + \vec{x}_h = \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix} + r\begin{pmatrix} -2 \\ 0 \\ 1 \end{pmatrix}$