Solution to Hwk 6

4.4 2)

a) 1 does not belong to span S

b) Span S consists of all vectors of the form $\begin{bmatrix} a \\ 0 \end{bmatrix}$, where a is any real number. Thus the vector $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ is not in Span S.

c) Span S consists of all vectors of M_{22} of the form $\begin{bmatrix} a & b \\ b & a \end{bmatrix}$, where a and b are any real numbers. Thus, the vector $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ is not in span S.

11) $A = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & 2 & 3 & 1 \\ 2 & 1 & 3 & 1 \\ 1 & 1 & 2 & 1 \end{bmatrix}$

After doing row operations $(AX = 0)$

A becomes $\left[\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \Rightarrow \begin{array}{l} x_4 = 0 \\ x_2 = -x_3 \\ x_1 = -x_3 \end{array}$

general solution:

$X = \begin{bmatrix} -r \\ -r \\ r \\ 0 \end{bmatrix}$ $\therefore$ The vector that spans the solution is $\begin{bmatrix} -1 \\ -1 \\ 1 \\ 0 \end{bmatrix}$

16) From exercise 43 in section 1.3, we have $Tr(AB) = Tr(BA)$ and $Tr(AB-BA) = Tr(AB) - Tr(BA) = 0$

hence, Span T is a subset of the set S of all $n \times n$ matrices with trace $= 0$. However, S is a proper subset of M_{nn}.

4-5.

b) The set S is linearly dependent.

12b)

$$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 2 \end{bmatrix}$$

$$a_1\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} + a_2\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} + a_3\begin{bmatrix} 0 & 1 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Corresponding system

$$a_1 + a_2 = 0$$
$$a_1 + a_3 = 0$$
$$a_1 = 0$$
$$a_1 + 2a_2 + 2a_3 = 0$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 2 & 2 & 0 \end{array}\right] \xrightarrow{\text{row operation}} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

This set S is linearly independent, because each column has a leading 1.

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c) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}, \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}, \begin{bmatrix} 2 & 2 \\ 1 & 1 \end{bmatrix}$

$$a_1\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} + a_2\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} + a_3\begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix} + a_4\begin{bmatrix} 2 & 2 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Corresponding system

$a_1 + 2a_2 + 3a_3 + 2a_4 = 0$
$a_1 + 3a_2 + a_3 + 2a_4 = 0$
$a_1 + a_2 + 2a_3 + a_4 = 0$
$a_1 + 2a_2 + a_3 + a_4 = 0$

$\Rightarrow$ Corresponding matrix

$$\left[\begin{array}{cccc|c} 1 & 2 & 3 & 2 & 0 \\ 1 & 3 & 1 & 2 & 0 \\ 1 & 1 & 2 & 1 & 0 \\ 1 & 2 & 1 & 1 & 0 \end{array}\right]$$

This can be changed to $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ by row operations

and since each column has a leading 1. This set S is linearly independent.

20) Suppose $a_1w_1 + a_2w_2 + a_3w_3 = a_1(v_1+v_2+v_3) + a_2(v_2+v_3) + a_3v_3 = 0$

Since $\{v_1, v_2, v_3\}$ is linearly independent, $a_1 = 0$, $a_1 + a_2 = 0$ and hence $a_2 = 0$. and also $a_1 + a_2 + a_3 = 0 \Rightarrow a_3 = 0$.

Thus $\{w_1, w_2, w_3\}$ is linearly independent.

24) Form the linear combination:

$$c_1 A v_1 + c_2 A v_2 + \cdots + c_n A v_n = A(c_1 v_1 + c_2 v_2 + \cdots c_n v_n) = 0$$

Since A is non singular. Theorem 2.9 implies that

$$c_1 v_1 + c_2 v_2 + \cdots + c_n v_n = 0$$

Since $\{v_1, v_2 \ldots v_n\}$ is linearly independent, we have $c_1 = c_2 = \cdots c_n = 0$. Hence, $\{Av_1, Av_2 \ldots Av_n\}$ is linearly independent.