§3.4.

2(a). $\operatorname{adj} A = \begin{pmatrix} 2 & -7 & -6 \\ 1 & -7 & -3 \\ -4 & 7 & 5 \end{pmatrix}$

(b) $\det A = -7$

7(b). $\begin{pmatrix} \frac{3}{14} & -\frac{3}{7} & \frac{1}{7} \\ \frac{1}{7} & \frac{5}{7} & -\frac{4}{7} \\ -\frac{1}{14} & \frac{1}{7} & \frac{2}{7} \end{pmatrix}$

9: $\frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

§3.5.

2. $\begin{cases} x_1 = 1 \\ x_2 = -1 \\ x_3 = 0 \\ x_4 = 2 \end{cases}$

6. $\begin{cases} x_1 = 1 \\ x_2 = \frac{2}{3} \\ x_3 = -\frac{2}{3} \end{cases}$

§4.1

12(b). $\vec{u} + \vec{v} = \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix}$

$2\vec{u} - \vec{v} = \begin{pmatrix} 3 \\ -4 \\ 11 \end{pmatrix}$

$3\vec{u} - 2\vec{v} = \begin{pmatrix} 4 \\ -7 \\ 18 \end{pmatrix}$

§4.1

12(b). $\vec{0} - 3\vec{v} = \begin{pmatrix} -3 \\ -6 \\ 9 \end{pmatrix}$

20: $c_1 = r$, $c_2 = s$, $c_3 = t$.

§4.2.

2. (a) No

(b) Yes

(c) $\vec{0} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

(d) Yes. If $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, with $a \cdot b \cdot c \cdot d = 0$, then $-A = \begin{pmatrix} -a & -b \\ -c & -d \end{pmatrix}$ with $(-a)(-b)(-c)(-d) = 0$.

(e) No. V is NOT closed under $\oplus$.

6. P is a vector space.
Let $p(t)$, $q(t)$ be polynomials not both zero. Suppose the larger of their degrees is n. Then $p(t) + q(t)$ and $c \cdot p(t)$ are computed as in example 6. The properties of definition 4.4 are verified as in Example 6 in section 4.2.

12. the zero vector is real number 1.
If u is a vector (that is, a positive real number), then "negative of u" is $\frac{1}{u}$.

§4.2.

13. The zero vector in V is the constant zero function.

17. No.

The zero element for $\oplus$ is the real number 1, but then $u=0$ has no negative. Thus (4) in definition 4.4 fails to hold.

21. (b). $c \odot \vec{0} = c \odot (\vec{0} \oplus \vec{0})$

$= (c \odot \vec{0}) \oplus (c \odot \vec{0})$ (*)

$\Rightarrow c \odot \vec{0} = 0$ (add "negative of $c \odot \vec{0}$" to both sides of (*))

(c). If $c=0$, statement is true.

If $c \neq 0$, then

$\frac{1}{c} \odot (c \odot \vec{u}) = \frac{1}{c} \odot \vec{0}$

$\Rightarrow (\frac{1}{c} \cdot c) \odot \vec{u} = \vec{0}$ (by part (b))

$\Rightarrow 1 \odot \vec{u} = \vec{0}$

$\Rightarrow \vec{u} = \vec{0}$.

25. If $a \odot \vec{u} = b \odot \vec{u}$, then

$(a-b) \odot \vec{u} = \vec{0}$

use (c) of Theorem 4.2, we get

$a-b=0 \Rightarrow a=b$.

§4.3

6 (d). NOT a subspace.

8 (b) NOT a subspace.

15. (a) is a subspace.

19. (c) is a subspace.

32. for any element $\begin{bmatrix} a & b \\ b & c \end{bmatrix}$, it can be written as

$\begin{pmatrix} a & b \\ b & c \end{pmatrix} = \begin{pmatrix} a & 0 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & b \\ b & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & c \end{pmatrix}$

$= a\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} + b\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + c\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$

$= a \cdot \vec{v}_1 + b\vec{v}_2 + c \cdot \vec{v}_3$.

34. (a) is a linear combination of $\vec{v}_1, \dots \vec{v}_4$.

(b) is NOT a linear combination of $\vec{v}_1, \dots \vec{v}_4$.

§4.3.

14. We have

$$A\mathbf{z} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} a+b \\ c+d \end{bmatrix},$$

so A is in W if and only if $a+b=0$ and $c+d=0$. Thus, W consists of all matrices of the form

$$\begin{bmatrix} a & -a \\ c & -c \end{bmatrix}.$$

Now if

$$A_1 = \begin{bmatrix} a_1 & -a_1 \\ c_1 & -c_1 \end{bmatrix} \quad \text{and} \quad A_2 = \begin{bmatrix} a_2 & -a_2 \\ c_2 & -c_2 \end{bmatrix}$$

are in W, then

$$A_1 + A_2 = \begin{bmatrix} a_1 & -a_1 \\ c_1 & -c_1 \end{bmatrix} + \begin{bmatrix} a_2 & -a_2 \\ c_2 & -c_2 \end{bmatrix} = \begin{bmatrix} a_1+a_2 & -(a_1+a_2) \\ c_1+c_2 & -(c_1+c_2) \end{bmatrix}$$

is in W. Moreover, if k is a scalar, then

$$kA_1 = k\begin{bmatrix} a_1 & -a_1 \\ c_1 & -c_1 \end{bmatrix} = \begin{bmatrix} ka_1 & -(ka_1) \\ kc_1 & -(kc_1) \end{bmatrix}$$

is in W. Alternatively, we can observe that every vector in W can be written as

$$\begin{bmatrix} a & -a \\ c & -c \end{bmatrix} = a\begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix} + c\begin{bmatrix} 0 & 0 \\ 1 & -1 \end{bmatrix},$$

so W consists of all linear combinations of two fixed vectors in M_{22}. Hence, W is a subspace of M_{22}.

29. Certainly $\{\mathbf{0}\}$ and R^2 are subspaces of R^2. If $\mathbf{u}$ is any nonzero vector then span $\{\mathbf{u}\}$ is a subspace of R^2. To show this, observe that span $\{\mathbf{u}\}$ consists of all vectors in R^2 that are scalar multiples of $\mathbf{u}$. Let $\mathbf{v} = c\mathbf{u}$ and $\mathbf{w} = d\mathbf{u}$ be in span $\{\mathbf{u}\}$ where c and d are any real numbers. Then $\mathbf{v}+\mathbf{w} = c\mathbf{u}+d\mathbf{u} = (c+d)\mathbf{u}$ is in span $\{\mathbf{u}\}$ and if k is any real number, then $k\mathbf{v} = k(c\mathbf{u}) = (kc)\mathbf{u}$ is in span $\{\mathbf{u}\}$. Then by Theorem 4.3, span $\{\mathbf{u}\}$ is a subspace of R^2.

To show that these are the only subspaces of R^2 we proceed as follows. Let W be any subspace of R^2. Since W is a vector space in its own right, it contains the zero vector $\mathbf{0}$. If $W \neq \{\mathbf{0}\}$, then W contains a nonzero vector $\mathbf{u}$. But then by property (b) of Definition 4.4, W must contain every scalar multiple of $\mathbf{u}$. If every vector in W is a scalar multiple of $\mathbf{u}$ then W is span $\{\mathbf{u}\}$. Otherwise, W contains span $\{\mathbf{u}\}$ and another vector which is not a multiple of $\mathbf{u}$. Call this other vector $\mathbf{v}$. It follows that W contains span $\{\mathbf{u}, \mathbf{v}\}$. But in fact span $\{\mathbf{u}, \mathbf{v}\} = R^2$. To show this, let $\mathbf{y}$ be any vector in R^2 and let

$$\mathbf{u} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}, \quad \mathbf{v} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}, \quad \text{and} \quad \mathbf{y} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}.$$

We must show there are scalars c_1 and c_2 such that $c_1\mathbf{u} + c_2\mathbf{v} = \mathbf{y}$. This equation leads to the linear system

$$\begin{bmatrix} u_1 & v_1 \\ u_2 & v_2 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}.$$

Consider the transpose of the coefficient matrix:

$$\begin{bmatrix} u_1 & v_1 \\ u_2 & v_2 \end{bmatrix}^T = \begin{bmatrix} u_1 & u_2 \\ v_1 & v_2 \end{bmatrix}.$$

This matrix is row equivalent to I_2 since its rows are not multiples of each other. Therefore the matrix is nonsingular. It follows that the coefficient matrix is nonsingular and hence the linear system has a solution. Therefore span $\{\mathbf{u}, \mathbf{v}\} = R^2$, as required, and hence the only subspaces of R^2 are $\{\mathbf{0}\}$, R^2, or scalar multiples of a single nonzero vector.