Solution to hwk 8

5.5)

10) Basis for null space of A: $\left\{ \begin{bmatrix} -1/3 \\ 7/3 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -7/3 \\ -2/3 \\ 0 \\ 1 \end{bmatrix} \right\}$

Basis for row space of A: $\left\{ \begin{bmatrix} 1 & 0 & 1/3 & 7/3 \end{bmatrix}, \begin{bmatrix} 0 & 1 & -7/3 & 2/3 \end{bmatrix} \right\}$

Basis for null space of AT: $\left\{ \begin{bmatrix} -1/2 \\ 1/2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1/2 \\ 3/2 \\ 0 \\ 1 \end{bmatrix} \right\}$

Basis for column space of A: $\left\{ \begin{bmatrix} 1 \\ 0 \\ 1/2 \\ 1/2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -1/2 \\ 1/2 \end{bmatrix} \right\}$

20) $v = \begin{bmatrix} 1 \\ 2 \\ -1 \\ 0 \end{bmatrix}$

$$w = \operatorname{Proj}_W v = (v, w_1)w_1 + (v, w_2)w_2 + (v, w_3)w_3 + (v, w_4)w_4$$

$\| v - \operatorname{Proj}_W v \| = \underline{\underline{2}}$

27) Let v be a vector in R^n. By theorem 5.12(a), the column space of A^T is the orthogonal complement of the null space of A. This means that R^n = null space of $A \oplus$ column space of A^T. Hence, there exist unique vectors w in the null space of A and v in the column space of A^T so $v = w + v$

5.b)

2) $A = \begin{bmatrix} 2 & 1 \\ 1 & 0 \\ 0 & -1 \\ -1 & 1 \end{bmatrix}$ $b = \begin{bmatrix} 3 \\ 1 \\ 2 \\ -1 \end{bmatrix}$

$\hat{x} = \begin{bmatrix} 24/17 \\ -8/17 \end{bmatrix} \simeq \begin{bmatrix} 1.4118 \\ -0.4706 \end{bmatrix}$

6.1)

10) (a)

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

(b) $\begin{bmatrix} r & 0 & 0 \\ 0 & r & 0 \\ 0 & 0 & r \end{bmatrix}$

c) $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

28) we have

$$L(f+g) = \int_a^b (f(x)+g(x))\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$$

$$= L(f) + L(g)$$

and $L(cf) = \int_a^b cf(x)\,dx = c\int_a^b f(x)\,dx = cL(f)$