Math 234: Final Review

Please, do these problems.
Chapter VII.8 number 3
Chapter VII.12 numbers 2, 5cgik
Chapter VII.17 numbers 3, 5, 7, 9, 11
and

1. Find the parametrization of the surface. There are many correct ways of doing this.

   (a) Tilted plane inside cylinder: The portion of the plane $x + y + z = 1$ inside the cylinder

   (i) $y^2 + z^2 = 9$

   (b) Circular cylinder band: The portion of the plane $(x - 2)^2 + z^2 = 4$ between the planes $y = 0$ and $y = 3$.

2. Use a parametrization to express the area of the surface as a double integral. Then evaluate the integral. Again, there is more than one correct way.

   (a) Sawed off sphere: The lower portion cut from the sphere $x^2 + y^2 + z^2 = 2$, by the cone $z = \sqrt{x^2 + y^2}$.

   (b) Parabolic band: The portion of the paraboloid $y = x^2 + z^2$ between $y = 2$ and $y = 5$.

3. Integrate $G(x, y, z) = x\sqrt{y^2 + 4}$ over the surface cut from the parabolic cylinder $y^2 + 4z = 16$ by the planes $x = 0$, $x = 1$ and $z = 0$.

4. Finding the flux across a surface. Use a parametrization to find the flux $\int\int_S \vec{F} \cdot \vec{n} dA$ across the surface in the given direction.

   (a) Sphere: $\vec{F} = zx\vec{i} + zy\vec{j} + z^2\vec{k}$ across the portion of the sphere $x^2 + y^2 + z^2 = a^2$ in the first octant in the direction away from the origin.

   (b) Paraboloid: $\vec{F} = 4x\vec{i} + 4y\vec{j} + 2\vec{k}$ outward (normal away from the z-axis) through the portion of the surface cut from the bottom of the paraboloid $z = x^2 + y^2$ by te plane $z = 1$.

5. Use Green's Theorem to compute the integral along the closed curve C, oriented in counterclockwise direction:

$$\oint_C xy\, dx + x^2\, dy$$

   where C is the line segment from $(-2, 0)$ to $(2, 0)$ and the top half of the circle $x^2 + y^2 = 4$.

6. Find a function f such that $\vec{F} = \vec{\nabla} f$. Then evaluate $\int_C \vec{F} \cdot d\vec{r}$, where

$$\vec{F} = \begin{pmatrix} 2xz + \sin y \\ x\cos y \\ x^2 \end{pmatrix}$$

$$C : r(\vec{t}) = \begin{pmatrix} \cos t \\ \sin t \\ t \end{pmatrix}, 0 \le t \le 2\pi$$

7. (a) Find a parametrization for the part of the plane $z = y - 3$ inside $x^2 + y^2 = 16$

   (b) Find the area of the surface $z = x + y^2$ above the triangle with vertices $(0,0), (1,1), (0,1)$.

8. Find the curl and the divergence of

$$\vec{F} = \begin{pmatrix} yz \\ y^2 + xz \\ xy \end{pmatrix}.$$

   Is $\vec{F}$ conservative?

9. Let f be a scalar function defined on a region in $\mathbb{R}^3$, let $\vec{F}$ be a vectorfield. State whether the expression is a number, a vectorfield or meaningless.

   $\vec{\nabla} \times f$

   $\vec{\nabla} \times \vec{F}$

   $\vec{\nabla}\vec{F}$

   $\vec{\nabla} \cdot (\vec{\nabla} f)$

   $\vec{\nabla} \times (\vec{\nabla} \times \vec{F})$

10. Find the flux of $\vec{F}$ across the surface S, where

$$\vec{F} = \begin{pmatrix} e^y \\ ye^x \\ x^2 y \end{pmatrix}$$

   and S is the part of the paraboloid $z = x^2 + y^2$ above the square $0 \le x \le 1$, $0 \le y \le 1$, with upward orientation.

11. Use Stokes' Theorem to evaluate

$$\int_C \vec{F} \cdot d\vec{x}$$

   where

$$\vec{F} = \begin{pmatrix} x^2 z \\ xy^2 \\ z^2 \end{pmatrix}$$

   and C is the curve of intersection of the plane $x + y + z = 1$ and the cylinder $x^2 + y^2 = 9$, oriented counterclockwise as viewed from above.

12. Use the Divergence Theorem to calculate the flux integral

$$\iint_S \vec{F} \cdot \vec{n}\, dA$$

   where

$$\vec{F} = \begin{pmatrix} 3xy \\ y^2 \\ -x^2 y^4 \end{pmatrix}$$

   and S is the surface of the tetrahedron with vertices $(0,0,0), (1,0,0), (0,1,0), (0,0,1)$ and oriented outward.