Definitions and Formulas 3

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Let A = accumulated balance after Y years
P = starting principal
APR = annual percentage rate (as a decimal)
n = number of compounding periods per year
Y = number of years (may be a fraction)
PMT = regular payment (deposit) amount
a = inflation rate (a decimal)
i = interest rate (a decimal)

Compound Interest Formula: $A = P(1 + \frac{APR}{n})^{nY}$

Annual Percentage Yield: APY $APY = (1 + \frac{APR}{n})^n - 1$

Formula for Continuous Compounding: $A = P * e^{APR*Y}$

Savings Plan Formula: $A = PMT * \frac{[(1+\frac{APR}{n})^{nY} - 1]}{\frac{APR}{n}}$

Total and Annual Return: $totalreturn = \frac{A-P}{P}$
$annualreturn = \left(\frac{A}{P}\right)^{(1/Y)} - 1$

Current Yield of a Bond: $\text{current yield} = \frac{\text{annual interest payment}}{\text{current price of bond}}$

Loan Payment Formula: $PMT = P * \frac{\frac{APR}{n}}{\left[1 - \left(1+\frac{APR}{n}\right)^{(-nY)}\right]}$

The CPI Formula $\frac{CPI_X}{CPI_Y} = \frac{price_X}{price_Y}$

The Present Value of principal P, Y years into the future, r=annual interest, a=inflation:
$$A = P * [\frac{1+r}{1+a}]^Y$$

Real Growth g: $g = \frac{r-a}{1+a}$

Real Growth over Y years: $g(Y) = [1 + \frac{r-a}{1+a}]^Y - 1$

Suppose $x_1, x_2, ...x_n$ are numeric data values, then
the mean is $\mu = \frac{x_1+x_2+...+x_n}{n}$.
the variance s^2 is $s^2 = \frac{(x_1-\mu)^2+(x_2-\mu)^2+...+(x_n-\mu)^2}{n-1}$.
The standard deviation s is the square root of the variance s^2.

The $68-95-99.7$ Rule for Normal Distributions:
68% of the observations fall within 1 standard deviation of the mean.
95% of the observations fall within 2 standard deviations of the mean.
99.7% of the observations fall within 3 standard deviations of the mean.

Quartiles of Normal Distributions:
Q_1 is .67 standard deviations below the mean
Q_3 is .67 standard deviations above the mean

For a simple random sample of size n,
the sample proportion of successes is $\hat{p} = \frac{\text{count of successes in the sample}}{n}$
The mean of the sampling distribution is$\hat{p}$
the standard deviation is $\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$.

The $68-95-99.7$ Rule applies here aswell.

A an event, and A^C its complement, $P(A^C) = 1 - P(A)$.

A, B independent events, then $P(A \text{ and } B) = P(A) * P(B)$.
A, B dependent events, then $P(A \text{ and } B) = P(A) * P(B \text{given } A)$.
A, B overlapping events, then $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$.
A, B nonoverlapping events, then $P(A \text{ or } B) = P(A) + P(B)$.
"at least once":
$P(\text{at least one event A in n trials}) = 1 - [P(A^C)]^n$.

Given sample space S with outcomes $x_1, x_2..., x_n$
with respective probabilities $p_1, p_2..., p_n$, then
the mean μ is: $\mu = x_1 * p_1 + x_2 * p_2 + ... + x_n * p_n$.
the variance σ^2 is: $\sigma^2 = (x_1-\mu)^2 * p_1 + (x_2-\mu)^2 * p_2 + ... + (x_n-\mu)^2 * p_n$
the standard deviation σ is the square root of the variance.

Central Limit Theorem:
Draw a simple random sample (SRS) of size n from any large population

with mean μ and standard deviation σ, then
the mean of the sampling distribution of $\bar{x}$ is μ.
the standard deviation of $\bar{x}$ is $\frac{\sigma}{\sqrt{n}}$.
the sampling distribution of $\bar{x}$ is approximately normal
when the sample size n is large.

Fairness Criteria

Majority Criterion:
If a candidate gets a majority ($> 50\%$) of the first place votes,
he/she should be winner.

Condorcet Criterion:
winning candidate should also be winner of pairwise comparisons.

Monotonicity Criterion:
Suppose X is the winner and suppose that in another election some voters are
able to rank X higher, with no change for the other candidates,
then X should still win.

Irrelevant Alternatives Criterion:
Suppose X is the winner, if one or more losing candidates drop
from the race, X should still be the winner.

The Adjusted Winner Procedure

each party distributes 100 points over the items in a way that reflects their
relative worth to that party.

Initially give each item to the party that assigns it more points.

Tally the point totals.

The party with the lower sum is given the items on which both parties placed the same number of points.

Tally the point totals.

If the point totals are not equal, transfer items (fractional items) from the winner to the loser by a special method until the point totals are equal.

Method:

go through the items on the winner's list and compute the point ratio:

Transfer (fractional a part of) items in order of increasing point ratio.

What part? The part that would make the point totals equal. Set up the equation
for the point totals and solve for x.

Knaster's Inheritance Procedure:

Each heir makes a simultaneous and independent bid for the asset.

However, now the winner has to pay out the losers according to their perceived fair share.

This leaves \$ 40,000 in the kitty, which is distributed equally to all heirs.

If there is more than one asset, use Knaster's Method one asset at a time.

Cake Division Procedures:

for 3 players: Bob, Carol, Ted

Bob divides the cake into 3 pieces: X, Y, Z

case 1: Carol approves of X
Ted approves of Y
give Z to Bob
case 2: Carol and Ted both approve of X, and both disapprove of Z
merge X and Y, then XY is greater than 2/3
let Carol and Ted do a divide and choose for 2 on XY
give Z to Bob