Problems due Thursday 9/21/2016. No email or web submissions.

You can discuss the homework with other students. You must cite your collaborators.

1. **Read** Drazin sections 1.*, 2.*, 5.3
2. Drazin Problem 5.11, 5.12, 5.13.
   - For 5.12, we discussed it in a different way in class, briefly explain/comment on the differences.
   - For 5.13, treat the general case of an arbitrary steady base flow $\boldsymbol{U}(\boldsymbol{x})$ but derive also the explicit Euler-Lagrange equations for a plane shear flow $\boldsymbol{U} = U(y)\hat{\boldsymbol{x}}$. Problem 5.13 presumes that you know some *Calculus of Variations*, ask, collaborate, research, if you do not.
3. Consider the non-dimensionalized Boussinesq equations

$$\boldsymbol{\nabla} \cdot \boldsymbol{v} = 0 \tag{1}$$

$$\partial_t \boldsymbol{v} + \boldsymbol{v} \cdot \boldsymbol{\nabla} \boldsymbol{v} + \boldsymbol{\nabla} p = \frac{1}{Re} \nabla^2 \boldsymbol{v} + T\,\hat{\boldsymbol{y}} \tag{2}$$

$$\partial_t T + \boldsymbol{v} \cdot \boldsymbol{\nabla} T = \frac{1}{Pe} \nabla^2 T \tag{3}$$

   for the velocity $\boldsymbol{v}(\boldsymbol{x}, t)$ and temperature $T(\boldsymbol{x}, t)$, with the boundary conditions $\boldsymbol{v} = 0$ and $T = \mp 1$ at $y = \pm 1$ with periodic boundary conditions in the horizontal x and z directions (periods L_x and L_z respectively), where $Re > 0$ is a *Reynolds* number and $Pe > 0$ is a *Péclet* number.

   - Show that $\boldsymbol{v} = 0$, $T = -y$ is a steady solution for all Re and Pe.
   - Let $\boldsymbol{v} = \boldsymbol{u}(\boldsymbol{x}, t)$ and $T = -y + \Theta(\boldsymbol{x}, t)$ and derive the evolution equations for the perturbation velocity $\boldsymbol{u}$ and temperature Θ. What are the boundary conditions for $\boldsymbol{u}$ and Θ?
   - Derive the evolution equation for the total perturbation 'energy' defined as

$$\frac{1}{2} \int_V \left(\boldsymbol{u} \cdot \boldsymbol{u} + \Theta^2 \right) dV$$

     where V is the domain of the flow and dV is the volume element. Use integration by parts to rewrite the diffusion terms in negative definite form. What is the production term?
   - Likewise but first rescale $\boldsymbol{u} = \sqrt{(Re/Pe)}\, \boldsymbol{u}'$ and use the 'energy'

$$\frac{1}{2} \int_V \left(\sqrt{\frac{Re}{Pe}}\, \boldsymbol{u}' \cdot \boldsymbol{u}' + \sqrt{\frac{Pe}{Re}}\, \Theta^2 \right) dV.$$

     What are the production and dissipation terms in this case? What does this suggest with respect to absolute stability of the base state? What is the appropriate variational problem to guarantee absolute stability?