1. Derive conservation of mass for a gas flowing down a tube of cross-sectional area $A(x)$ where x is the direction along the axis of the tube. Do this in two ways (1) from first principles assuming that the variables (mass density and gas velocity) are uniform over a given cross-section and (2) by integration of the general two-dimensional equation $\rho_t + \boldsymbol{\nabla} \cdot (\rho \boldsymbol{v}) = 0$ over the cross-sectional area (height) $A(x)$. Compare the results.

2. Find a bounded self-similar solution to the equation $u_t = u_{xxx}$.

3. What is the Green's function for the heat equation in $\mathbb{R}^3$?

4. Consider the heat equation $u_t = u_{xx}$ in $x > 0$, $t > 0$, with boundary conditions $u_x = -|h|u$ at $x = 0$, u bounded as $x \to \infty$ and initial condition $u(x, 0) = \cos kx$, k real. Is this problem well-posed? What can you find out about its solution?

5. Show that $[H(x + t) - H(x - t)]\, \delta'(t) = -2\delta(x)\delta(t)$.

6. What is the polar coordinate representation of $\delta(x)\delta(y)$? (x, y in $\mathbb{R}$).

7. Solve the equation $xu'' + u' = f(x)$ for $0 < x < 1$ with $u(1) = 0$ and $\lim_{x \to 0^+} u$ bounded using a Green's function.

8. Find the general solution to $(\partial_t + a\partial_x)(\partial_t + b\partial_x)u = 0$ with x in $\mathbb{R}$ and $u(x, 0) = f(x)$, $u_t(x, 0) = g(x)$.