This is a Take-Home exam. You must work on this alone. You may use your class notes, the Zauderer book and the 703 book (Strang, Introduction to Applied Mathematics) but no other book. You may work on this for as long as you want but must turn it in to VV 819 on Fri May 11 by 11:11 am as we all agreed.
Please use Fourier transform definitions that were used in class and are summarized in the handout posted on the web page `http://kleene.math.wisc.edu/~waleffe/M704/`

1. Find the Fourier transforms of (a) $|x|H(x)$ and (b) $f(x)$ if $f(x) = f(x+P)$ (Hint: use Fourier *Series*).

2. Consider a general mass-spring system $m\ddot{x} + Kx = 0$, where $x(t)$ is the displacement from the rest position and $\ddot{x} = d^2x/dt^2$. In general, the spring stiffness K is a function of x, $K = K(x)$. Consider small oscillations of the mass. Find the change of variables (both x and t) that transforms the equation into $\ddot{u} + u = \epsilon u^2 + C\epsilon^2 u^3 + O(\epsilon^3)$, where $\epsilon \ll 1$ is a (small and positive) constant, C is a constant and $u(t)$ is a function with magnitude $O(1)$ ("order 1"). Find the angular frequency of the oscillation up to $O(\epsilon^2)$.

3. A long time ago, we found a self-similar solution of the heat equation $u_t = \kappa u_{xx}$, i.e. a solution of the form $u(x,t) = t^\alpha f(xt^\beta)$, thereby reducing the PDE to an ODE for the function $f(\eta)$. Another important type of self-similar solution corresponds to a *traveling wave* solution, i.e. a solution of the form $u(x,t) = f(x - Vt)$ for some (real) constant V where we require that u be bounded as $|x| \to \infty$. Show that there are no traveling wave solutions for the heat equation but that there are traveling wave solutions for the nonlinear equation $u_t + uu_x = \kappa u_{xx}$. In the latter case, assume that $u \to u_1$ as $x \to -\infty$ and $u \to u_2$ as $x \to +\infty$, where u_1 and u_2 are constants.