Fourier Transform: Unbounded x space $\leftrightarrow$ unbounded k space

$$\begin{aligned} u(x) &= \int_{-\infty}^{\infty} \hat{u}(k)\, e^{ikx} dk \\ \hat{u}(k) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} u(x)\, e^{-ikx} dx \end{aligned} \tag{1}$$

Mathematicians prefer to symmetrize these formulas by defining $\hat{u}(k) = \hat{v}(k)/\sqrt{2\pi}$, but Physicists and Applied Mathematicians usually prefer the form above because they often look at single modes or interaction of a few modes.

Fourier Series: Bounded, periodic x space $\leftrightarrow$ unbounded, discrete k space: $k_m = 2m\pi/L$

$$\begin{aligned} u(x) &= \sum_{m=-\infty}^{\infty} \tilde{u}_m\, e^{ik_m x} \\ \tilde{u}_m &= \frac{1}{L} \int_{-L/2}^{L/2} u(x)\, e^{-ik_m x} dx \end{aligned} \tag{2}$$

The wavenumber is now quantized because each mode must be periodic of period L, $e^{ikx} = e^{ik(x+L)}$, or $e^{ikL} = 1$. You can also write these formulas in a form that tends to the Fourier transform as $L \to \infty$ by defining $\tilde{u}_m = 2\pi \tilde{v}_m / L$.

Fourier Series: Unbounded, discrete x space: $x_j = j\Delta x \leftrightarrow$ bounded, periodic k space

$$\begin{aligned} u_j &= \int_{-\pi/\Delta x}^{\pi/\Delta x} \tilde{u}(k)\, e^{ikx_j} dk \\ \tilde{u}(k) &= \frac{\Delta x}{2\pi} \sum_{j=-\infty}^{\infty} u_j\, e^{-ikx_j} \end{aligned} \tag{3}$$

where j is an integer. These are really the same formulas as those above, except that x and k are permuted. The domain of periodicity ($\equiv L$ above) in k space is $2\pi/\Delta x$.

Discrete Fourier Transform: Discrete, bounded x space $\leftrightarrow$ discrete, bounded k space

$$\begin{aligned} u_j &= \sum_{m=-N/2}^{N/2-1} \tilde{u}_m\, e^{ik_m x_j} \\ \tilde{u}_m &= \frac{1}{N} \sum_{j=-N/2}^{N/2-1} u_j\, e^{-ik_m x_j} \end{aligned} \tag{4}$$

where m, j are integers. The wavenumbers $k_m = 2m\pi/L$ as before and $x_j = j\Delta x$, with the extra requirement now that $N\Delta x = L$. Note that $u_j = u_{j+N}$ and $\tilde{u}_m = \tilde{u}_{m+N}$, thus the summation bounds can be shifted to $0, N-1$ instead of $-N/2, N/2-1$.