1. Find the maximum of z subject to the constraints $x + y + z = 1$ and $z^2 = 3xy^3$.

2. What is the area of the triangle whose vertices are the three points $P = (1, 4, 3)$, $Q = (2, 0, -1)$ and $R = (0, 0, 5)$? Find an equation in the form $F(x, y, z) = 0$ for the plane containing the three points. Can you also find a parametric representation for the plane in the form $x = x(u, v)$, $y = y(u, v)$ and $z = z(u, v)$?

3. A hiker is climbing Boring mountain whose elevation is $z = 1000 - 2x^2 - 3y^2$. Find the trajectory of the climber if she climbs as steeply as possible. Express the trajectory in the form $y = f(x)$, $z = g(x)$. What will be the distance travelled by the hiker from her starting point at (x_0, y_0, z_0) to the summit? (express your answer as an integral of a function of x, $f(x)$ over x. Specify $f(x)$ and the limits of integration. You do not need to evaluate the integral).

4. (a) Show how the divergence theorem implies that $\oint_S \phi \vec{n}\, dS = \int_V \vec{\nabla} \phi\, dV$ where S is the closed surface enclosing the volume V and $\vec{n}$ is the outward unit normal.

(b) If the pressure field $p(x, y, z)$ (= force per unit area) is given as $p = p_0 - \rho g z$, where p_0, ρ and g are constants, calculate the total pressure force exerted on a body of volume V. You must justify your answer. (Hint: the pressure exerts a force per unit area on the surface of the body in the inward normal direction).

5. Calculate the work performed by the force $\vec{F} = y^2\, \hat{\mathbf{i}}$ acting on a particle that went from $x = 0$ to $x = 4$ along the x-axis then back along the half circle of radius 2 centered at $x = 2$, $y = 0$. (Hint: works = force X distance)

6. (a) What is the image of the strip $-\infty < x < \infty$, $0 \le y \le \pi$ under the mapping

$$w = \frac{1 + e^z}{1 - e^z}?$$

Identify characteristic points and boundaries and their image in the w-plane.

(b) Find the steady temperature distribution $T(x, y)$ in the upper half-plane when the temperature on the lower boundary is $T(x, 0) = 1$ when $|x| < 1$ and $T(x, 0) = 0$ when $|x| > 1$ (and $T(x, y) \to 0$ as $x^2 + y^2 \to \infty$). The heat equation, with t as time and κ as the heat conductivity, is

$$\frac{\partial T}{\partial t} = \kappa \nabla^2 T.$$

7. (a) Calculate the integral $\oint_C \frac{dz}{z^2 - 2z}$, where C is the counterclockwise unit circle $|z| = 1$.

(b) Calculate $\int_0^{2\pi} \frac{d\theta}{2 - \sin\theta}$.

8. Find all the roots of $z^4 = 1 + i$. Express your answers in polar form.