The Biot-Savart law stated at `http://en.wikipedia.org/wiki/Biot-Savart_law` reads

$$d\vec{\boldsymbol{B}} = \frac{\mu_0 I}{4\pi}\frac{d\vec{\boldsymbol{\ell}} \times \hat{\boldsymbol{r}}}{r^2} = \frac{\mu_0 I}{4\pi}\frac{d\vec{\boldsymbol{\ell}} \times \vec{\boldsymbol{r}}}{r^3} \tag{1}$$

where μ_0 is the magnetic constant and I is constant by conservation of charge. Equation (1) states that the little bit of current $Id\vec{\boldsymbol{\ell}}$ at some location $\vec{\boldsymbol{\ell}}$ in space creates the little bit of magnetic field $d\vec{\boldsymbol{B}}$ at the point "$\vec{\boldsymbol{r}}$" where this "$\vec{\boldsymbol{r}}$" is the position vector *from the line current*. The magnetic field at point P only cares about where the current is, not where you the big O-bserver are located! Makes sense.

From the arbitrary origin O, the current element is at location C, say, with $\overrightarrow{OC} = \vec{\boldsymbol{\ell}}$ and the magnetic field is calculated at point P so the "$\vec{\boldsymbol{r}}$" in that wikipedia Biot-Savart law is really "$\vec{\boldsymbol{r}}$"$=\overrightarrow{CP} = \overrightarrow{OP} - \overrightarrow{OC} = \vec{\boldsymbol{r}} - \vec{\boldsymbol{\ell}}$, with our usual $\vec{\boldsymbol{r}} = \overrightarrow{OP}$. So (1) written more explicitly is

$$d\vec{\boldsymbol{B}} = \frac{\mu_0 I}{4\pi}\frac{d\vec{\boldsymbol{\ell}} \times \overrightarrow{CP}}{|\overrightarrow{CP}|^3} = \frac{\mu_0 I}{4\pi}\frac{d\vec{\boldsymbol{\ell}} \times (\vec{\boldsymbol{r}} - \vec{\boldsymbol{\ell}})}{|\vec{\boldsymbol{r}} - \vec{\boldsymbol{\ell}}|^3} \tag{2}$$

This is the bit of magnetic field at point $\vec{\boldsymbol{r}}$ induced by the bit of current $Id\vec{\boldsymbol{\ell}}$ located at $\vec{\boldsymbol{\ell}}$. Formula (2) is scarier looking, but more precise.

That extra precision is crucial in order to compute the magnetic field induced by a complete current. If the current flows around a (closed) wire $\mathcal{C}$ then the total magnetic field at point $\vec{\boldsymbol{r}}$ induced by that current is $\vec{\boldsymbol{B}}(\vec{\boldsymbol{r}}) = \oint_{\mathcal{C}} d\vec{\boldsymbol{B}}$ which is

$$\vec{\boldsymbol{B}} = \frac{\mu_0 I}{4\pi}\oint_{\mathcal{C}} \frac{d\vec{\boldsymbol{\ell}} \times (\vec{\boldsymbol{r}} - \vec{\boldsymbol{\ell}})}{|\vec{\boldsymbol{r}} - \vec{\boldsymbol{\ell}}|^3} \tag{3}$$

This is why we need that $\vec{\boldsymbol{\ell}}$ and $\vec{\boldsymbol{r}}$. We need to sum over all current elements, $Id\vec{\boldsymbol{\ell}}$, located at $\vec{\boldsymbol{\ell}}$. There are an "infinity" of them around the loop $\mathcal{C}$. We're looking at the sum of their magnetic fields induced at point $\vec{\boldsymbol{r}}$. That point is arbitrary, but fixed as far as the sum over current elements is concerned.

Now we want to calculate (3) in the special case where the loop is a straight wire from $-\infty$ to $+\infty$. Let's pick our z axis in the direction of that current so $\vec{\boldsymbol{\ell}} = \ell\hat{\boldsymbol{z}}$, $d\vec{\boldsymbol{\ell}} = d\ell\hat{\boldsymbol{z}}$ and the integral (3) becomes

$$\vec{\boldsymbol{B}} = \frac{\mu_0 I}{4\pi}\oint_{\mathcal{C}} \frac{d\vec{\boldsymbol{\ell}} \times (\vec{\boldsymbol{r}} - \vec{\boldsymbol{\ell}})}{|\vec{\boldsymbol{r}} - \vec{\boldsymbol{\ell}}|^3} = \frac{\mu_0 I}{4\pi}\int_{-\infty}^{\infty} \frac{\hat{\boldsymbol{z}} \times (\vec{\boldsymbol{r}} - \ell\hat{\boldsymbol{z}})}{|\vec{\boldsymbol{r}} - \ell\hat{\boldsymbol{z}}|^3}d\ell \tag{4}$$

This integral can be simplified as follows:

$$\begin{aligned}
\int_{-\infty}^{\infty} \frac{\hat{\boldsymbol{z}} \times (\vec{\boldsymbol{r}} - \ell\hat{\boldsymbol{z}})}{|\vec{\boldsymbol{r}} - \ell\hat{\boldsymbol{z}}|^3}d\ell &= (\hat{\boldsymbol{z}} \times \vec{\boldsymbol{r}})\int_{-\infty}^{\infty} \frac{d\ell}{|\vec{\boldsymbol{r}} - \ell\hat{\boldsymbol{z}}|^3} \\
&= (\hat{\boldsymbol{z}} \times \vec{\boldsymbol{r}})\int_{-\infty}^{\infty} \frac{d\ell}{\left(x^2 + y^2 + (z-\ell)^2\right)^{3/2}} \\
&= (\hat{\boldsymbol{z}} \times \vec{\boldsymbol{r}})\int_{-\infty}^{\infty} \frac{d\ell}{\left(\rho^2 + (z-\ell)^2\right)^{3/2}} \\
&= \frac{\hat{\boldsymbol{z}} \times \vec{\boldsymbol{r}}}{\rho^2}\int_{-\infty}^{\infty} \frac{du}{(1+u^2)^{3/2}}
\end{aligned} \tag{5}$$

where $\rho = \sqrt{x^2+y^2} = |\hat{\boldsymbol{z}} \times \vec{\boldsymbol{r}}|$ is the distance to the z axis, and I've made the change of variables $\ell \to u = (\ell - z)/\rho$ so $du = d\ell/\rho$ to calculate the integral. The cool thing about this change of variable is that we're just left with a *parameter-free* integral, a constant in other words. So we have indeed shown that the magnetic field induced by an infinite straight current flowing in the $\hat{\boldsymbol{z}}$ direction has the form

$$\vec{\boldsymbol{B}} = C\,\frac{\hat{\boldsymbol{z}} \times \vec{\boldsymbol{r}}}{|\hat{\boldsymbol{z}} \times \vec{\boldsymbol{r}}|^2} \tag{6}$$

for some constant C, as used in our math 321 notes and examples. Hurrah!! Life is good. Although you might want to add an argument that the u integral actually exists so that C is *not* infinity!

If you're good at Math 221 and Math 222 integration, you can figure out that integral using a substitution: $u = \tan\theta$ with $-\pi/2 \le \theta \le \pi/2$ and $du = d\theta/(\cos^2\theta) = (1+\tan^2\theta)\,d\theta$, so

$$\int_{-\infty}^{\infty} \frac{du}{(1+u^2)^{3/2}} = \int_{-\pi/2}^{\pi/2} \cos\theta \, d\theta = 2, \tag{7}$$

so that mysterious $C = \dfrac{\mu_0 I}{2\pi}$.