This is a TAKE HOME EXAM **due Tue Oct 30, 11:00am at regular lecture**.
I, the undersigned, understand that I must work on this ALONE, I cannot collaborate with anyone or ask anyone for help. I can use the lecture notes, my course notes, any other book and even the internet, however I MUST acknowledge explicitly which book and/or website I used. I understand that I may be asked to present my solutions orally to the Prof and/or TA during a discussion or at other suitable time and that my final grade would be decided by my oral presentation. My write-up should be clean and well organized. I understand that the grader can give me NO credit, even for correct work, if my write-up was too difficult for the grader to decypher, in the opinion of the grader.

SIGNATURE and DATE:

PRINTED NAME:

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1. Consider the curve $\vec{\boldsymbol{r}}(\theta) = \vec{\boldsymbol{r}}_c + a\cos\theta\,\vec{\boldsymbol{e}}_1 + b\sin\theta\,\vec{\boldsymbol{e}}_2$ where

$$\vec{\boldsymbol{e}}_1 = \sin\frac{\pi}{3}\cos\frac{\pi}{4}\vec{\boldsymbol{e}}_x + \sin\frac{\pi}{3}\sin\frac{\pi}{4}\vec{\boldsymbol{e}}_y + \cos\frac{\pi}{3}\vec{\boldsymbol{e}}_z$$

and

$$\vec{\boldsymbol{e}}_2 = \cos\frac{5\pi}{3}\frac{\vec{\boldsymbol{e}}_z\times\vec{\boldsymbol{e}}_1}{|\vec{\boldsymbol{e}}_z\times\vec{\boldsymbol{e}}_1|} + \sin\frac{5\pi}{3}\frac{\vec{\boldsymbol{e}}_1\times(\vec{\boldsymbol{e}}_z\times\vec{\boldsymbol{e}}_1)}{|\vec{\boldsymbol{e}}_z\times\vec{\boldsymbol{e}}_1|}$$

where $\vec{\boldsymbol{e}}_x$, $\vec{\boldsymbol{e}}_y$ and $\vec{\boldsymbol{e}}_z$ are the unit vectors in the mutually orthogonal cartesian directions x, y, z, respectively. $\vec{\boldsymbol{r}}_c$ is a constant vector and a and b are real numbers.

(a) Show that $\vec{\boldsymbol{e}}_1$ and $\vec{\boldsymbol{e}}_2$ satisfy $\vec{\boldsymbol{e}}_i\cdot\vec{\boldsymbol{e}}_j = \delta_{ij}$.

(b) Show that $\vec{\boldsymbol{r}}(\theta)$ corresponds to a canonical implicit equation for an ellipse. You must carefully explain and define everything.

(c) Explain why this is a closed curve.

(d) Derive explicit θ integrals for the length and area of this closed curve.

(e) Calculate the area exactly.

(f) Show that the length $\mathcal{L}$ is a function of the form $\mathcal{L} = a\,f(a/b)$.

(g) Calculate the length approximately for $a/b = 2$ if your last name starts with a letter in A-G, $a/b = 3$ for H-Q, $a/b = 4$ for R-Z. Take several, successive approximations and show that your approximations are converging. What is you best answer and how many digits do you trust? You must show your work. The best solution would be to provide a small code (e.g. in matlab) to do this calculation for various levels of approximation but this is not required and explicit numerical results must still be provided.

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2. The pressure outside the sphere of radius R centered at $\vec{\boldsymbol{r}}_c$ is $p(\vec{\boldsymbol{r}}) = p_0 + A\vec{\boldsymbol{r}}\cdot\vec{\boldsymbol{e}}$ where $\vec{\boldsymbol{e}}$ is a fixed unit vector and p_0 and A are constants. Calculate the net force on the sphere defined as $\vec{\boldsymbol{F}} = -\int_S p\,d\vec{\boldsymbol{S}}$ with $d\vec{\boldsymbol{S}}$ pointing outward. *You must do a direct calculation of the surface integral, you cannot use the divergence theorem.*