FW Math 321 **Exam 1, T Oct 11, 2005**

NO BOOK, NO NOTES, NO CALCULATOR. *Use clean and precise notation, distinguish between vectors and scalars, write your dots in dot products and your crosses in cross products. Draw clean sketches (with a few words of explanation) and use a few words ("given", "hence", ...) to lay out your reasoning. READ CAREFULLY!*

NAME and Major:

---

1. Use vectors to show that the 3 lines dropped from the vertices of a triangle perpendicularly to the opposite side intersect at the same point.

2. Given two vectors $\boldsymbol{v}$ and $\boldsymbol{B}$, *not of unit norm*, give vector formula for the *vector* components of $\boldsymbol{v}$ parallel and perpendicular to $\boldsymbol{B}$. Draw a sketch.

3. The angle between vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ is φ. The angle between the unit normal $\boldsymbol{n}$ to the plane $\boldsymbol{a}$, $\boldsymbol{b}$ and vector $\boldsymbol{c}$ is θ (where $\boldsymbol{a}$, $\boldsymbol{b}$, $\boldsymbol{n}$, in that order, is right handed). Express $\det(\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c})$ and $\det(\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{n})$ in terms of the magnitudes of $\boldsymbol{a}$, $\boldsymbol{b}$, $\boldsymbol{c}$ and the angles φ and θ.

4. What is the distance between the point with cartesian coordinates (x_0, y_0, z_0) and the plane defined by the cartesian equation $ax + by + cz = d$, where (x, y, z) are the cartesian coordinates of a point on the plane and a, b, c, d are known real numbers?
(i) Draw a sketch and recast this problem in vector notation specifying the key vectors in terms of the data; (ii) Obtain a vector solution, *then* plug in the data to get a final explicit answer in terms of the given data.

5. The following problems use the convention of *summation over repeated indices.* Each index as the range $(1, 2, 3)$. Calculate
(a) δ_{ii}, (b) $\delta_{ij}\delta_{jk}\delta_{kl}\delta_{lm}$, (c) $\epsilon_{ijk}\delta_{jk}$, (d) State precisely what $\epsilon_{ijk}a_j b_k$ represents,
(e) decypher then provide a formula for $\epsilon_{ijk}a_j(\epsilon_{klm}b_l c_m)$ that does not have any ϵ's in it.

6. When a particle of mass m and electric charge q moves through a magnetic field $\boldsymbol{B}$, it experiences the Lorentz force $\boldsymbol{F} = q(\boldsymbol{v}\times\boldsymbol{B})$, where $\boldsymbol{v}$ is the velocity of the particle. If $\boldsymbol{B}$ is constant, use Newton's law of motion to show that the particle moves on a circular cylinder. More precisely, show that the particle motion consists of the superposition of a uniform motion in one direction and a uniform rotation in the perpendicular directions. What is the angular rotation frequency? (angular rotation rate).

7. The components of three vectors $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$ with respect to the orthonormal basis $\boldsymbol{e}_1$, $\boldsymbol{e}_2$, $\boldsymbol{e}_3$ are (a_1, a_2, a_3), (b_1, b_2, b_3) and (c_1, c_2, c_3), respectively. It turns out that

$$a_1^2 + a_2^2 + a_3^2 = b_1^2 + b_2^2 + b_3^2 = c_1^2 + c_2^2 + c_3^2 = 1, \quad \text{and}$$

$$a_1 b_1 + a_2 b_2 + a_3 b_3 = b_1 c_1 + b_2 c_2 + b_3 c_3 = c_1 a_1 + c_2 a_2 + c_3 a_3 = 0.$$

(a) What geometric information does this tell us about $\boldsymbol{a}$, $\boldsymbol{b}$, $\boldsymbol{c}$?
(b) Can $\boldsymbol{a}$, $\boldsymbol{b}$, $\boldsymbol{c}$ be used as a basis?
(c) If so, what are the components of $\boldsymbol{e}_1$, $\boldsymbol{e}_2$, $\boldsymbol{e}_3$ with respect to the $\boldsymbol{a}$, $\boldsymbol{b}$, $\boldsymbol{c}$ basis?
(d) Is $\boldsymbol{a}$, $\boldsymbol{b}$, $\boldsymbol{c}$ right-handed?