**1.** $\begin{vmatrix} 0 & a & b & c & d \\ -a & 0 & a & b & c \\ -b & -a & 0 & a & b \\ -c & -b & -a & 0 & a \\ -d & -c & -b & -a & 0 \end{vmatrix} = 0$ because $\det(A) = det(A^T) = \det(-A) = (-1)^n \det(A)$ where $n$ is the matrix dimension, $n = 5$ here.

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**2.** Solve $z^2 = \dfrac{a - ia}{b + ib}$, $\forall$ (for all) $a, b > 0$ and complex $z$.
$z^2 = \dfrac{a-ia}{b+ib} = \dfrac{a}{b}\dfrac{1-i}{1+i} = \dfrac{a}{b}\dfrac{e^{-i\pi/4}}{e^{i\pi/4}} = \dfrac{a}{b}e^{-i\pi/2+2ik\pi}, \Rightarrow z = \sqrt{\dfrac{a}{b}}e^{-i\pi/4+ik\pi} = \pm\sqrt{\dfrac{a}{b}}e^{-i\pi/4} = \pm\sqrt{\dfrac{a}{2b}}(1-i)$.

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**1.** $\det(\boldsymbol{a}\boldsymbol{b}^T) = 0$, because each column is a multiple of $\boldsymbol{a}$ (each row is a multiple of $\boldsymbol{b}^T$ also), so by the shearing and stretching rule, det=0.

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**2.** Find all complex $z$'s such that $e^z = -\pi$. Any complex $z$ can be written in cartesian or polar form: $z = x + iy = re^{i\theta+2ik\pi}$. Here: $e^z = e^x e^{iy}$ and $-\pi = \pi e^{i\pi+2ik\pi}$, and $e^z = -\pi$ implies

$$e^x = \pi \Rightarrow x = \ln \pi, \quad y = (2k+1)\pi.$$

So there are an infinite number of solutions, one for every integer $k = 0, \pm1, \pm2, \ldots$ $e^z = e^{x+iy}$ is a periodic function of $y$ of period $2\pi$.