This is a TAKE-HOME EXAM. You must work on this **alone, by yourself, with no one's help besides your own**. *You can use the book (Edwards and Penney), your class and homework notes but no other reference material. So any information obtained from the Internet in any way is also forbidden, except of course copies of this exam. You may use your calculator. You can work on this for as long as you want but must state how long it took you and turn it in by the* **due date and time: 1:20 pm, Monday Sept 30, 2002**.

YOUR NAME:

TIME for completion:

1. Solve

$$x(x+1)y' = \sin x - xy, \qquad y(0) = 1.$$

Your answer may contain integral(s) but all constants and limits of integration must be specified. Estimate $y(\pi)$.

2. Make a beautiful hand-sketch (no computer plot!) of solution curves of $x^2y' + y = 1$. What can you say about $\lim_{x\to 0} y(x)$? about $\lim_{x\to+\infty} y(x)$? about the solution with initial condition $y(0) = y_0 \neq 0$?
Extra credit: same questions but for $x^2y' + y = x$.

3. An initially empty cylindrical tank of constant cross-section A is filled with water at the constant rate k. The water drains from a hole of cross-section a at the bottom of the tank under the action of gravity. Assume Torricelli's law for the water velocity at the hole.

(a) Draw a sketch of this problem, identifying all relevant variables.

(b) Derive the differential equation governing the evolution of the water depth h as a function of time t.

(c) Determine the water depth as $t \to \infty$, explaining briefly your answer (assume the tank is deep enough that it does not overflow).

(d) Find a definite integral expression relating h and t. Integrate.

3. Edwards and Penney 2.1.20.
4. Edwards and Penney 2.3.20.

5. The mass m_e of an electron is much less than the mass M_p of a proton, so as a first approximation we can assume that the proton does not move during the interaction of an electron and a proton (just like we neglect the effect of a space shuttle on the earth). Assume that the interaction force between the proton and the electron is $F(r) \equiv a/r^2 - b/r^4$ where r is the distance from the center of the proton to the center of the electron, a and b are positive constants and the force is attractive if $F(r) > 0$, repulsive if $F(r) < 0$.

(a) Derive the equation that governs the radial velocity $v \equiv dr/dt$ of the electron, where t is time.

(b) What is the equilibrium position of the electron?

(c) Is that equilibrium stable or unstable?

(d) What is the escape velocity for an electron located at the equilibrium position?

6. Use elimination to answer Edwards and Penney 3.2.28. Make sure you clearly identify what operations you are performing on the equations to eliminate variables (for instance by writing E1 - 4 E2 $\equiv \cdots$, etc.)