

MATHLAB and MATLAB 8

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1. The goal of this session is to find approximations to the eigenvalues and eigenfunctions for ODE eigenvalue problems.

2. Consider first an example where the solution is already known:

$$-y'' = ry, \quad y(0) = 0 = y(\pi),$$

r being the eigenvalue parameter. The eigenvalues are $r_n = n^2$ and the eigenfunctions are multiples of $\sin nx$.

3. Write the equation as a system:

$$x_1' = x_2, \quad x_2' = -rx_1$$

and enter it as a function M-file called e.g. ex5.m:

```
function xpr = ex5(t,x);
global r;
xpr = zeros(2,1);
xpr(1) = x(2);
xpr(2) = - r * x(1);
```

The command ‘global r ’ means that r is a parameter whose value will be entered when we start to compute.

4. Set up a script M-file, ev5.m, that solves the system with IC $x_1(0) = 0$, $x_2(0) = 1$ and graphs the first component of the solution:

```
[t,x] = ode45 ('ex5', [0,pi], [0,1]);
plot (t, x(:,1))
```

The syntax for ode45 may be slightly different depending on the version of MATLAB your computer has.

5. If r is an eigenvalue, $x_1(\pi) = 0$. Moreover by the Sturm-Liouville theory, if $r = r_1$, $x_1(t)$ is positive in $(0, \pi)$; if $r = r_2$, $x_1(t)$ has exactly 1 zero in $(0, \pi)$; if $r = r_3$, $x_1(t)$ has exactly 2 zeroes in $(0, \pi)$, etc. We will try by varying r to make $x_1(\pi) = 0$ and $x_1(t) > 0$ in $(0, \pi)$ thereby obtaining (approximately) r_1 and its corresponding eigenvector. Similarly r_2 , etc. can be obtained.

6. To carry out this process and find r_1 , type e.g.

```
global r
r = 11;    ev5
```

The graph of $x_1(t)$ that appears has 3 zeroes in $(0, \pi)$. Next try

```
r = 21; ev5
```

Now the graph of $x_2(t)$ has 4 zeroes in $(0, \pi)$. This suggests that r be decreased to find r_1 . In fact, we know $r_1 = 1$ and $r_k = k^2$. Vary r to check out this fact.

7. Employ the above method and the fact that r_1 is about 4 to find r_2, r_3, r_4 to e.g. the nearest integer for the equation

$$(*) \quad -y'' + (1 + x^2)y = ry, \quad y(0) = 0 = y(\pi)$$

Print and hand in the graphs of the corresponding eigenfunctions, indicating the values of r_2, r_3, r_4 on your graphs.

8. As you will have noticed in carrying out 7, estimating when $x_1(\pi) = 0$ by eyeball is of limited accuracy. A better way to proceed is to use the **fzero** command in MATLAB. It finds a zero of a given function, e.g. $g(s)$ in a given interval $[a, b]$ provided that g has opposite signs at a and b . To use it to calculate e.g.

r_5 for (*), use of the above method shows $21 < r_5 < 25$. Set up a new function M-file, *evm*:

```
function z = ev(s)
global r;
r = s;
[t,x] = ode45('ex6', [0,pi], [0,1]);
z = x(end, 1)
```

Note 'ex 6' is the function M-file for (*). Also $x(\text{end}, 1)$ denotes the last entry in the computation and hence corresponds to $x_1(\pi)$. If you have an older version of MATLAB you may have to replace $x(\text{end}, 1)$ by $x(\text{length}(t), 1)$. The file *evm* calculates $x_1(\pi)$ for each given value of r .

Now in the command window type:

```
fzero('ev', [21, 25])
```

and you'll get $r_5 = 29.3461$. Use the same method to get r_6 and r_7 and print out the graphs of the respective eigenvectors.