

Mathlab and MATLAB 5

1. The goal of this session is to develop some tools for treating systems of ODE's.

First we learn how to plot a vector function $x(t) = (x_1(t), x_2(t))$ in the x_1, x_2 plane. (This is also called a phase plane plot). This can be most directly as follows. To plot $x(t) = (\sin t, \cos t)$ over $0 \leq t \leq 2\pi$, type

```
t = 0 : 0.05 : 2 * pi;
```

```
plot(sin(t), cos(t))
```

2. A three dimensional plot can be obtained by using the above statement and

```
plot 3 (sin(t), cos(t), t)
```

or

```
plot 3 (t, sin(t), cos(t))
```

3. Plot $x(t) = (\sin t, \cos \sqrt{2}t)$ for $0 \leq t \leq 12\pi$ in the phase plane and in 3 dimensions. Print and hand in your graphs.

4. MATLAB has several ODE solvers built into it. We will use the one called ode45. It is based on a variable step Runge-Kutta method. The syntax depends on whether you have a newer or older version of MATLAB. To solve the equations or system

$$x' = \text{ex2}(t, x)$$

with IC $x(t_0) = x_0$ on the interval $t_0 \leq t \leq t_f$, use the syntax

```
[t, x] = ode45('ex2', t0, tf, x0);
```

for the old version or

```
[t, x] = ode45('ex2', [t0, tf], x0);
```

for the new one. (Type `help ode45` to check whether you have the old or new version on your machine). The equation will be given by a function *M*-file.

5. To illustrate, for the system

$$x_1' = x_2 - x_1^2, \quad x_2' = -x_1 - 2x_1x_2$$

with $x_1(0) = 0, x_2(0) = 1$ on $0 \leq t \leq 10$, set up an *M*-file called e.g. `ex3` using `xpr` for x' . The *M*-file is then

function `xpr = ex3(t,x);`

`xpr = zeros (2,1); %this tells MATLAB xpr is a col. 2 vector`

`(or 2 by 1 matrix)`

$$\text{xpr}(1) = x(2) - x(1)^2;$$

$$\text{xpr}(2) = -x(1) - 2 * x(1) * x(2);$$

Now solve by typing (for the new version).

$$[t, x] = \text{ode45}('ex3', [0, 10], [0, 1]);$$

Note that the IC are entered as the vector $[0, 1]$. You can plot x_1 and x_2 as functions of t by typing

$$\text{plot}(t, x)$$

If you only want to see e.g. the graph of x_1 , type

$$\text{plot}(t, x(:, 1))$$

To get a phase plane plot, type

$$\text{plot}(x(:, 1), x(:, 2))$$

6. The equation $y'' + 25y = \sin 5.6t$ exhibits 'beats'. Writing the equation as a first order system, solve the IVP on $0 \leq t \leq 45$ with IC

$$(a) \quad y(0) = 0 = y'(0) \qquad (b) \quad y(0) = 0, y'(0) = 1$$

(c) $y(0) = 1, y'(0) = 0$ (d) $y(0) = 1 = y'(0)$

Note if the system is $x' = f(t, x)$, then $x_1(t) = y(t)$. Print and hand in a plot of $(t, y(t))$ for (a)-(d).

7. For the IVP, $y'' + 0.5y' + 16y = 100 \sin t$, $y(0) = 0 = y'(0)$,

(a) Plot the solution for $0 \leq t \leq 50$

(b) Find analytically the periodic solution, $z(t)$, of this equation and plot the difference between $z(t)$ and $y(t)$ for $0 \leq t \leq 50$.