

MATHLAB and MATLAB 4

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1. The goal of this session is to show how to use MATLAB to help solve higher order linear equations with constant coefficients.
2. The first step is to find the roots of a polynomial $p(x) = s^n + a_1 s^{n-1} + \cdots + a_n$.
Type

$$\mathbf{p} = [1 \quad \mathbf{a}_1 \quad \mathbf{a}_2 \quad \mathbf{a}_3 \quad \dots \quad \mathbf{a}_n];$$

(thereby entering the coefficients of p as a row vector). E.g. do this for $p(s) = s^2 + 3s + 2$. Then type

$$\mathbf{r} = \text{roots}(\mathbf{p})$$

to get the roots.

3. Find the general solution of
 - (a) $y^{(5)} + 5y^{(4)} + 4y^{(3)} + 3y^{(2)} + 2y' + y = 0$.
 - (b) $y^{(10)} + 7y^{(7)} + y^{(3)} - 17y = 0$.

Hand in your answer.

4. Trying to solve the IVP leads to a system of linear equations. E.g. suppose the general solution of a 4th order equation turns out to be

$$y(t) = c_1 e^{2t} + c_2 e^{-4t} + c_3 e^{-t} + c_4 e^{3t}$$

and you have IC: $y(0) = 3$, $y'(0) = -3$, $y'''(0) = 21$ and $y'''(0) = -57$. Using the IC leads to the system of equations

$$y(0) = 3 = c_1 + c_2 + c_3 + c_4$$

$$y'(0) = -3 = 2c_1 - 4c_2 - c_3 + 3c_4$$

$$y''(0) = 21 = 4c_1 + 16c_2 + c_3 + 9c_4$$

$$y'''(0) = -57 = 8c_1 - 64c_2 - c_3 + 27c_4.$$

The coefficient matrix is

$$A = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 2 & -4 & -1 & 3 \\ 4 & 16 & 1 & 9 \\ 8 & -64 & -1 & 27 \end{pmatrix}.$$

Letting $c = \begin{pmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{pmatrix}$ and $b = \begin{pmatrix} 3 \\ -3 \\ 21 \\ -57 \end{pmatrix}$, the system of equations takes the form

$Ac = b$. This can be solved easily using MATLAB. Enter the matrix A via

$$A = [1 \ 1 \ 1 \ 1; 2 \ -4 \ -1 \ 3; 4 \ 16 \ 1 \ 9; 8 \ -64 \ -1 \ 27];$$

and b (as a column vector) via

$$b = [3; -3; 21; -57];$$

Then type

$$c = A \backslash b$$

to get the solution vector $c = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix}$.

4. (a) Solve the IVP

$$y^{(5)} - 2y^{(4)} - y' + 2y = 0,$$

$$y(0) = 4, y'(0) = -1, y''(0) = 2, y^{(3)}(0) = -5, y^{(4)}(0) = 4.$$

- (b) Graph the solution over the interval $[0, 6\pi]$ and hand in the solution and its graph.

5. (a) Solve the IVP

$$y^{(4)} - 4.6y^{(3)} + 7.91y^{(2)} - 6.026y' + 1.716y = 0,$$

$$y(0) = -1, y'(0) = -2, y''(0) = 3, y^{(3)}(0) = 5.$$

- (b) Graph the solution over $[-3, 1]$ and hand in the solution and its graph.

6. Solve the IVP

$$D^2(D - 1)^3y = 0,$$

$$y(0) = 0, y'(0) = 1, y''(0) = 0, y^{(3)}(0) = 0, y^{(4)}(0) = 0$$

and hand in your answer.