

Mathlab and MATLAB 2

- I. The purpose of this session is to introduce you to the software 'dfield'. At the MATLAB prompt, type

dfield5

and hit the return key. (The earlier version of the command is `dfield` so try this if your computer does not respond to `dfield5`). After a brief wait a new window labelled DFIELD Setup appears. The default equation is

$$(1) \quad x' = x^2 - t$$

and it has a corresponding range of values of t and x as indicated in the display window.

- II. Click on Proceed. You get a direction field for (1). To get the solution of (1) with IC $x(2) = 1$, move the mouse to the point $(2, 1)$ and click. You get the solution curve first calculated forward and then backward in time. Calculate the solution through a few more points.
- III. To get the solution curve through a given point more accurately than using the mouse as in II, click on DField Options, scroll to Keyboard Input and click. Then you can enter the initial data here with the aid of the mouse and click on Compute to get the solution curve.
- IV. To print your output, click on Print.
- V. Return to the Set Up window (e.g. by clicking on Window, scrolling to DField Setup, and clicking). Enter the equation $x' = x^2 - t^2$ with $t \in [-5, 5]$ and $x \in [-5, 5]$. Use dfield to find the solution curves through $(t, x) = (0, 0), (-3, 0), (0, 1), (4, 0)$. Print your output.
- VI. Do the same for $x' = 1 - t^2 + \sin tx$ with IC $x(0) = -3, -1, 0, 2, 4$. Find a good display window by experimentation and print your output.
- VII. The logistics equation is

$$x' = rx \left(1 - \frac{x}{K}\right).$$

For more sophisticated models, r and K may depend on time. Suppose $r = 1$ and

(a) $K(t) = 1$

(b) $K(t) = 1 - \frac{1}{2}e^{-t}$

(c) $K(t) = 1 + t$

(d) $K(t) = 1 - \frac{1}{2} \cos 2\pi t$ (In MATLAB, use pi for π).

(a) is the standard logistics model. In (b) and (c), K is monotone increasing. It is bounded for (b) and unbounded for (c). Such a K may arise in models of human populations where e.g. technological improvements increase the saturation population. In (d), K is periodic; it models populations where seasonal effects are taken into consideration. For (a), whatever IC, you choose $x(t) \rightarrow 1$ as $t \rightarrow \infty$. This is not the case for (b) - (d). However for each of (b) - (d), all IC lead to the same asymptotic behavior. Determine it qualitatively for each case by choosing several IC and examining the corresponding solutions. Hand in your graphs writing down your assessment of the asymptotic behavior. (Choose appropriate window sizes for each case with t and x nonnegative.)

VIII. You can use the “zoom in” feature of DField Options to get better pictures of the asymptotic states.