

PROBLEM 1:

A simple electric circuit consists of an inductance L , a resistance R and a voltage source $V(t)$. When a current $I(t)$ flows through the circuit, a voltage drop of RI occurs across the resistance and LdI/dt across the inductance, where t is time and L and R are constants, independent of time. These voltage drops are balanced by the voltage source which drives the current. Write down the differential equation for $I(t)$. Next, consider the case of alternating current $V(t) = V_0 \cos \omega t$, where the amplitude V_0 and the pulsation (or frequency) ω are constants. What are V_0 and ω in the US? It seems that this problem is defined by 4 parameters L , R , V_0 and ω . That is true but the qualitative behavior actually depends on only ONE nondimensional parameter. Instead of using t as the independent variable, use $\tau = \omega t$ and instead of I consider $\mathcal{I} = \omega LI/V_0$. Rewrite the equation in terms of τ and $\mathcal{I}$. What are the units of τ and $\mathcal{I}$? There should be only one parameter left in your equation, what is that parameter and what are its units? Call the parameter α and solve your equation for $\mathcal{I}(\tau)$ with initial condition $\mathcal{I}(0) = \mathcal{I}_0$. What is $\mathcal{I}(\tau)$ after a “long” time? Give a suitable estimate for “long time” in terms of the parameters of the problem, in other words for a given circuit, how long would you have to wait to observe that long time behavior?

Other nine problems are from Boyce and DiPrima: 2.2.22, 23, 38; 2.3.23; 2.4.10, 24; 2.5.10, 27; 2.6.18;