

*This is a TAKE-HOME EXAM. You should solve it on your own. You can use the textbook (Stewart), your notes and a calculator. You can work on this for as long (or as little) as you want. Your solutions must be turned in to your TA at your Wednesday Dec 1st discussion. NO LATE EXAMS ACCEPTED. Your write-up must be clean, convincing and as brief as possible without impairing clarity (Don't turn in scratch work. Show the important steps needed to go from the problem to its solution).*

1. A snowboarder slides (without friction) from  $(0,0)$  to  $(\pi R, -2R)$ . This takes time  $\pi\sqrt{R/g}$  if the track is a cycloid (as we calculated in class). Calculate how long it takes to slide between those two points (starting from  $(0,0)$  with no velocity) if the track is (a) a straight line, (b) a sinusoid  $y = a \sin(bx)$ , (c) a parabola  $x = cy^2$ . You may have to use Maple to evaluate some integrals. Rate the tracks from fastest to slowest. [Hint: energy is conserved so this means  $v^2/2 + gy = \text{Const.}$ , where  $g$  is the acceleration of gravity,  $v$  is the velocity of the snowboarder and  $y$  is pointing up, in the direction opposite to the gravitational force].
2. Four ants are chasing each other. Each one starts at one of the corner of a square of length  $L$  and chases the ant initially located at the next corner in the counterclockwise direction. The ants crawl at the same constant speed and always pointing directly towards the ant they are following. Draw a clean sketch of this problem. Symmetry and the directional information should allow you to deduce the differential equation that governs the trajectory of one of the ants. (Re)-Write the differential equation in polar coordinates. Find the equation for the trajectory of one of the ants. Sketch the trajectory. How far will an ant have travelled by the time it finally catches up with the ant it is following?
3. Stewart 10.2 # 70.
4. Calculate  $\sin 31^\circ$  (sine of 31 degrees) to 6 decimal places using only the 4 basic operations  $+$ ,  $-$ ,  $*$ ,  $/$  (i.e. not simply using the "sin" key on your calculator, but you can use the  $+$ ,  $-$ ,  $*$ ,  $/$  keys, as well as  $\pi$ ). Explain how you proceeded and how you can be sure of your 6 digits.