FW Math 222

TAKE-HOME EXAM 2 Due Wed Oct 20 1999

This is a TAKE-HOME EXAM. You should solve it on your own. You can use the textbook (Stewart), your notes and a calculator. You can work on this for as long (or as little) as you want. Your solutions must be turned in to your TA at your Wednesday Oct 20 discussion. NO LATE EXAMS ACCEPTED. Your write-up must be clean, convincing and as brief as possible without impairing clarity (Don't turn in scratch work. Show the important steps needed to go from the problem to its solution).

1. Use both the trapezoidal and midpoint rules to calculate

$$\int_0^\pi \frac{\sin x}{x} \, dx.$$

Is this an improper integral? Explain any practical problem you encounter. We'd like to know the answer to at least 3 digits. Explain how you proceeded to obtain an answer that you can trust up to 3 digits at least.

2. Devise a method to calculate

$$\int_{-1}^{1} \frac{e^x}{\sqrt{1-x^2}} \, dx.$$

Give a value for that integral as well as an estimate of the accuracy of that value (i.e. how many digits do you believe and why). You must explain the method you used and how you decided on how many digits to trust. Just providing an answer is not enough.

- **3.** Solve x(x+1)y' y = x(x+1) for x > 0 with y(1) = A. Show how you solved it.
- 4. Solve $y' = e^{x-y}$ with y(0) = 1. Show how you solved it.

5. $y(x) = 3e^{-x^2} + \int_1^x e^{u^2 - x^2} \cos u^3 \, du$ is the solution to a first order linear differential equation. What differential equation? You must show that y(x) indeed satisfies the diff eq that you propose. Also give an explicit "initial condition" for that y(x).

6. We have seen briefly that the equation

$$\ddot{\theta} + \frac{g}{L}\sin\theta = 0$$

governs the oscillations of a pendulum, where θ is the angle between the pendulum and the vertical, g is the acceleration of gravity and L is the length of the pendulum (see problem 36 in 7.8 for a figure). Multiply the equation by $\dot{\theta}$ and use the chain rule to integrate the equation once (Hint: $2\dot{\theta}\ddot{\theta} = d(\dot{\theta})^2/dt$, $\dot{\theta}\sin\theta = df(\theta)/dt$, for what $f(\theta)$?). The pendulum is released from angle $\theta_0 \neq 0$ with no initial velocity. Express the constant of integration in terms of those initial conditions. What is the first order equation for θ ? Which particular type of differential equation is it? (among those that we studied in 15.1 and 15.2). Give a definite integral for the time it takes the pendulum to swing from its initial position at θ_0 to the vertical position $\theta = 0$. Is your integral proper or improper? If it is improper, show that it converges.