1.

(a) $\frac{d}{dx}\int_1^8 \frac{\cos(\ln x)}{x^3}\,dx = d/dx(constant) = 0$

(b) $\int_0^{\pi/2} \frac{d}{dx}\left(\sin x\,\cos\frac{x}{2}\right)dx = \left[\sin x\,\cos\frac{x}{2}\right]_0^{\pi/2} = 1/\sqrt{2}$

(c) Derivative of $\int_0^v \cos x^3 dx = \cos v^3$

(d) Derivative of $\int_{3x}^{\pi x^2} \sin(u^2)du = \sin(\pi x^2)^2 d(\pi x^2)/dx - \sin(3x)^2 d(3x)/dx = 2\pi x\sin(\pi^2x^4) - 3\sin(9x^2)$

(e) "the antiderivative of $f(x)$ that is equal to π at $x = 3$" is $\int_3^x f(s)ds + \pi$.

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2. (a)

$$I(t) = I_0\, e^{-Rt/L} + \frac{1}{L}\int_0^t V(s)e^{(R/L)(s-t)}ds, \tag{2}$$

$$= I_0e^{-\alpha t} + \frac{e^{-\alpha t}}{L}\int_0^t V(s)e^{\alpha s}ds = e^{-\alpha t}\left(I_0 + \frac{1}{L}\int_0^t V(s)e^{\alpha s}ds\right),$$

where $\alpha = R/L$ to simplify the notation. Then, the product rule, $(uv)' = u'v + uv'$, and the fundamental theorem give

$$\frac{dI}{dt} = -\alpha I(t) + \frac{e^{-\alpha t}}{L}V(t)e^{\alpha t} = -\alpha I(t) + \frac{V(t)}{L}$$

Putting it all together (remembering that $\alpha = R/L$)

$$L\frac{dI}{dt} + RI = -RI(t) + V(t) + RI(t) = V(t).$$

Done with (a).
(b) $V(t) = V_0\sin\omega t$, then

$$I(t) = I_0e^{-\alpha t} + \frac{e^{-\alpha t}V_0}{L}\int_0^t \sin(\omega s)\, e^{\alpha s}ds$$

we do the integral using two integration by parts. Exactly like Example 4 in Sect. 7.1, p. 419 with a tad more algebra. We get

$$\int_0^t \sin(\omega s)\, e^{\alpha s}ds = \left[\frac{e^{\alpha s}}{\alpha^2+\omega^2}(\alpha\sin\omega s - \omega\cos\omega s)\right]_0^t$$
$$= \frac{e^{\alpha t}}{\alpha^2+\omega^2}(\alpha\sin\omega t - \omega\cos\omega t) + \frac{\omega}{\alpha^2+\omega^2}$$

Putting it all together

$$I(t) = I_0e^{-\alpha t} + \frac{e^{-\alpha t}V_0}{L}\frac{\omega}{\alpha^2+\omega^2} + \frac{V_0}{L(\alpha^2+\omega^2)}(\alpha\sin\omega t - \omega\cos\omega t).$$

(c) for long times $e^{-\alpha t} \to 0$ as $t \to \infty$ (if $\alpha > 0$ which it is, negative resistance and/or negative inductance do not exist in this world), so a "long time" after flipping the switch

$$I(t) \approx \frac{V_0}{L(\alpha^2 + \omega^2)}(\alpha \sin \omega t - \omega \cos \omega t),$$

this is simple alternating current. Note that it is not in phase with the voltage with is $V_0 \sin \omega t$. $I(t)$ has a $\cos \omega t$. Indeed using the trig formula $\sin(a - b) = \sin a \cos b - \sin b \cos a$, let $\cos \phi = \alpha/\sqrt{\alpha^2 + \omega^2}$ and $\sin \phi = \omega/\sqrt{\alpha^2 + \omega^2}$ i.e. $\tan \phi = \omega/\alpha$ then that last formula can be written

$$I(t) \approx \frac{V_0}{\sqrt{R^2 + L^2\omega^2}} \sin(\omega t - \phi).$$

(d) "Long times" means $\alpha t \gg 1$ (i.e. αt "much bigger than 1"), so $\alpha t \approx 10$ is pretty long. Indeed $e^{(-10)} \approx 4.5\ 10^{-5}$ (or $4.5E$–5 in scientific notation). So long times means $t \gg L/R$ ($t > 10L/R$ say). This varies from circuit to circuit.

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3. (see #38 p. 421) $I_n \equiv \int_0^\pi \cos^n x dx$. Then $I_n = 0$ if n odd because $\int_0^{\pi/2} \cos^n x dx = -\int_{\pi/2}^{\pi} \cos^n x dx$ (plot $\cos x$ in $[0, \pi]$). So focus on n even, i.e. $n = 2k$. Let's use integration by parts, exactly as in example 6, Sect. 7.1, p. 420, except that we have a cos instead of a sin. Small change. The relevant formula is actually given to you in exercise 38 p. 421:

$$\begin{aligned} I_n \equiv \int_0^\pi \cos^n x dx &= \frac{1}{n}\left[\cos^{n-1} x \sin x\right]_0^\pi + \frac{n-1}{2}\int_0^\pi \cos^{n-2} x\, dx \\ &= \frac{n-1}{n}\int_0^\pi \cos^{n-2} x\, dx \end{aligned}$$

Now if we use this same formula for $\int_0^\pi \cos^{n-2} x\, dx$ we get

$$I_n \equiv \int_0^\pi \cos^n x dx = \frac{n-1}{n}\frac{n-3}{n-2}\int_0^\pi \cos^{n-4} x\, dx$$

OK, let's keep going then, with n even,

$$I_n \equiv \int_0^\pi \cos^n x dx = \frac{(n-1)(n-3)...(3)(1)}{n(n-2)...(4)(2)}\int_0^\pi \cos^0 x\, dx = \pi \frac{(n-1)(n-3)...(3)(1)}{n(n-2)...(4)(2)}.$$

This is the answer when n is even. We already knew it's zero when n is odd but we can check it using this recurrence as well. If $n = 2k + 1$ then after k integration by parts we end up with $\int_0^\pi \cos x dx = 0$ multiplying everything.

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4. (see #54 p. 422) You can do this in 2 ways but first *make some good sketches!*. Hard to see what's going on without a good sketch.
(1) cylinder of radius π, height $\ln \pi$ MINUS all the cylinder slices (disks) of radius $y = e^x$ and width dx for $x = 0$ to $\ln \pi$, i.e.

$$V = \pi(\pi^2)\ln \pi - \int_0^{\ln \pi} \pi(e^x)^2 dx = ... = \pi^3 \ln \pi - \frac{\pi^3}{2} + \frac{\pi}{2}.$$

(2) cylindrical shells of radius y of width dy and height $x(y) = \ln y$, i.e.

$$V = \int_a^\pi 2\pi y\, x(y) dy = 2\pi \int_1^\pi y \ln y dy.$$

You do this last integral by parts ($u = \ln y$, $v' = y$, ...) and obtain same answer as with method (1), of course.

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5. (see #58 p. 428) (a) $V(t) = 110 \sin 100\pi t$. Period: $T = 1/50$, so we have 50 cycles/second. (b) $V_{rms}^2 = \frac{1}{T}\int_0^T [V(t)]^2 dt = 50 \times 110^2 \int_0^{1/50} \sin^2 100\pi t\ dt$, let $100\pi t = x$, $100\pi dt = dx$ so $V_{rms}^2 = \frac{50\times 110^2}{100\pi}\int_0^{2\pi} \sin^2 x dx = \frac{110^2}{2}$, so $V_{rms} = 110/\sqrt{2}$. See Section 6.5 for background about averages.

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6. (see #18 p. 428) $\displaystyle\int_0^{\pi/4} \frac{dx}{1-\sin x} = \int_0^{\pi/4} \frac{1+\sin x}{1-\sin^2 x}\, dx = \int_0^{\pi/4} \frac{1}{\cos^2 x} dx + \int_0^{\pi/4} \frac{\sin x}{\cos^2 x} dx =$ $[\tan x]_0^{\pi/4} + [1/\cos x]_0^{\pi/4} = \sqrt{2}$.

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7. (see #28 p. 443) $\displaystyle\int_0^1 \frac{x}{x^2+4x+4} dx$. This is a proper rational function (no need to divide) (1) expand denominator in product of simple factors, i.e. find roots of quadratic. Pretty obvious here: $x^2+4x+4 = (x+2)^2$, so we have a repeated root. This is Case II in 7.4. The partial fraction expansion is

$$\frac{x}{x^2+4x+4} = \frac{A}{x+2} + \frac{B}{(x+2)^2} = \frac{Ax+2A+B}{(x+2)^2},$$

thus $A = 1$ and $B = -2A = -2$. So

$$\int_0^1 \frac{x}{x^2+4x+4} dx = \int_0^1 \frac{dx}{x+2} - 2\int_0^1 \frac{1}{(x+2)^2} dx = [\ln|x+2|]_0^1 + 2\left[\frac{1}{x+2}\right]_0^1 = \ln\frac{3}{2} - \frac{1}{3}.$$

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8. (see # 54 p. 443) $\displaystyle\int_0^{\pi/2} \frac{\sin x \cos^2 x}{5+\cos^2 x} dx$. Let $w = \cos x$ then $dw = -\sin x dx$ and

$$I \equiv \int_0^{\pi/2} \frac{\sin x \cos^2 x}{5+\cos^2 x} dx = \int_0^1 \frac{w^2}{5+w^2} dw.$$

We now have a rational function but the deg(numerator)=deg(denominator) so we need to divide first. Easy here as $w^2 = (w^2+5) - 5$. So

$$\int_0^1 \frac{w^2}{5+w^2} dw = \int_0^1 dw - \int_0^1 \frac{5}{5+w^2} dw.$$

The first integral is trivial, the second is an arctan:

$$I = 1 - \sqrt{5}\left[\arctan\frac{w}{\sqrt{5}}\right]_0^1 = 1 - \sqrt{5}\arctan\frac{1}{\sqrt{5}}.$$