FW Math 222 **TAKE-HOME EXAM 1 Due Wed Sept 15 1999**

This is a TAKE-HOME EXAM. You should solve it on your own. You can use the textbook (Stewart), your notes and a calculator. You can work on this for as long (or as little) as you want but you must TIME YOURSELF and report how much time you spent. The amount of time you spent will not affect your grade. Your solutions must be turned in to your TA at your Wednesday Sept 15 discussion. NO LATE EXAMS ACCEPTED. Write briefly but clearly and show your steps but we do not need to see all your missteps... don't turn in scratch work.

1. Evaluate the following expressions:

(a) $\dfrac{d}{dx}\displaystyle\int_1^8 \frac{\cos(\ln x)}{x^3}\,dx$ (b) $\displaystyle\int_0^{\pi/2} \frac{d}{dx}\left(\sin x\,\cos\frac{x}{2}\right)dx$

(c) Derivative of $\displaystyle\int_0^v \cos x^3 dx$ (d) Derivative of $\displaystyle\int_{3x}^{\pi x^2} \sin(u^2)du$

(e) Write as a definite integral + constant: "the antiderivative of $f(x)$ that is equal to π at $x = 3$."

2. The electric current $I(t)$ in a simple electric circuit consisting of an inductance L, a resistance R and a voltage source $V(t)$ is governed by the *differential equation*

$$L\frac{d}{dt}I + RI = V, \tag{1}$$

where t is time and L and R are constants, independent of time. We will soon learn that the solution of this differential equation is

$$I(t) = I_0\, e^{-Rt/L} + \frac{1}{L}\int_0^t V(s)e^{(R/L)(s-t)}ds, \tag{2}$$

where I_0 is the current at time $t = 0$.
For now, you are asked to (a) verify that (2) is indeed a solution of (1), (b) find $I(t)$ explicitly when $V(t)$ is alternating current, i.e. $V(t) = V_0 \sin\omega t$, where V_0 and ω are constants, (c) what is the current $I(t)$ after a long time? (d) what does "long time" mean here (specify in terms of the physical parameters of the circuit: L and R).

3. Give an explicit formula for $\int_0^\pi \cos^n x dx$ (i.e. a formula that does not contain any integral).

4. Calculate the volume generated by the rotation about the x-axis of the region bounded by $y = e^x$, $x = 0$ and $y = \pi$.

5. Electricity is supplied in the form of alternating current that varies from -110 Volts to 110 Volts with a frequency of 50 Hz (cycles/second). (a) What is an equation for the voltage $V(t)$, where t is time in seconds? Voltmeters read the RMS (root-mean-square) voltage, which is the square root of the average value of $[V(t)]^2$ over one cycle. (b) What is the RMS voltage of the alternating current?

6. Calculate $\displaystyle\int_0^{\pi/4} \frac{dx}{1-\sin x}$.

7. Calculate $\displaystyle\int_0^1 \frac{x}{x^2+4x+4}dx$.

8. Calculate $\displaystyle\int_0^{\pi/2} \frac{\sin x\cos^2 x}{5+\cos^2 x}dx$.