1. [9.1 #26]

(1) $x = \sin 3u$, $y = \sin 4u$ is such that $|x|, |y| \leq 1$ so the plot is bounded and has to be either I, II or V. At $u = 0$, $x = y = 0$ so it has to be V.

(2) $x = u^4 - u^2$, $y = u + \ln u$ is such that $y \to -\infty$ as $u \to 0^+$ in which case $x \to 0$. The only curve for which this is true is VI.

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2. [9.1 # 36] Find parametric equations for the set of all points P determined as shown in the figure so that $|OP| = |AB|$. Sketch the curve.
Let θ be the angle between the x-axis and the radial line OB, then $|OB| = 2a/\cos\theta$ and $|OA| = 2a\cos\theta$ so $|OP| = |OB| - |OA|$. So in polar coordinates:

$$|OP| = r = 2a\left(\frac{1}{\cos\theta} - \cos\theta\right) = 2a\frac{\sin^2\theta}{\cos\theta},$$

or in Cartesian coordinates

$$x = 2a\sin^2\theta, \qquad y = 2a\frac{\sin^3\theta}{\cos\theta}.$$

As $\theta \to \pi/2$ we see that $y \to \infty$ and $x \to 2a$, so the curve asymptotes to the vertical $x = 2a$. For $\theta \ll 1$, the first terms of the Taylor series expansion of $\sin\theta$ and $\cos\theta$ give $x \approx 2a\theta^2$, $y \approx 2a\theta^3$ so $\dfrac{y}{2a} \approx \left(\dfrac{x}{2a}\right)^{3/2}$, near $x, y = 0$.

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3. [9.5 # 9] $r^2 = 4\cos 2\theta$

Need $\cos 2\theta \geq 0$, so θ is restricted to the ranges $-\pi/4 \leq \theta \leq \pi/4$ and $3\pi/4 \leq \theta \leq 5\pi/4$ (up to a factor of 2π). These two ranges in fact give portions of the curve that are mirror images across the y-axis. Furthermore, $r \leq 2$, so the curve lives inside the disk of radius 2 and in the angular sectors $-\pi/4 \leq \theta \leq \pi/4$ and $3\pi/4 \leq \theta \leq 5\pi/4$. Now $r(\pi/4) = r(-\pi/4) = 0$ so the curve intersects itself at the origin. The curve looks like a figure 8 lying on its side. The area it encloses is

$$A = 2\int_{-\pi/4}^{\pi/4} \frac{r^2}{2} d\theta = 4\int_{-\pi/4}^{\pi/4} \cos 2\theta d\theta = 4.$$

In Cartesian coordinates: $r^2 = 4\cos 2\theta = 4(\cos^2\theta - \sin^2\theta)$ so $r^4 = 4r^2\cos^2\theta - 4r^2\sin^2\theta$, i.e.

$$(x^2 + y^2)^2 = 4x^2 - 4y^2.$$

or $x^2 = y^2 + (x^2 + y^2)^2/4$ and near $x = y = 0$, where the curve intersect itself, $x^2 \approx y^2$ or $x \approx \pm y$, so the two tangents at the interesection have slopes ± 1. Another way to get the slopes: $x = r\cos\theta$, $y = r\sin\theta$, so near $\theta = \pi/4$ we have $\cos\theta \approx \cos\pi/4 = 1/\sqrt{2}$ and $\sin\theta \approx \sin\pi/4 = 1/\sqrt{2}$ and $x \approx y$. Similarly, near $\theta = -\pi/4$, $x \approx -y$.

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4. (a) $1,\quad \dfrac{1}{1+1},\quad \dfrac{1}{1+\dfrac{1}{1+1}},\quad \dfrac{1}{1+\dfrac{1}{1+\dfrac{1}{1+1}}},\ldots,$

this is the sequence $a_1 = 1$, $a_{n+1} = \dfrac{1}{1+a_n}$. If it converges the limit must satisfy $L = \dfrac{1}{1+L}$, so $L = (\sqrt{5}-1)/2$.

(b) $\displaystyle\lim_{n\to\infty} \frac{1+2^3+3^3+4^3+\cdots+n^3}{n^4} = 1/4$

(c) $\displaystyle\sum_{n=0}^{\infty}(-2)^n$, diverges. a_n does NOT $\to 0$ as $n \to \infty$. This is a geometric series with $q = (-2)$ and $|q| > 1$.

(d) $\displaystyle\sum_{n=0}^{\infty} \frac{3^n}{n!} = e^3$.

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5. Starting with a square with sides of length L join the middle of each edge to create a new square, then join the middle of the edges of that new square to create another and so on **indefinitely**. What is the sum of the areas of all the squares? (no $\sum$ in your answer).
The first square has area L^2. The next square has half the area, the third has half the area of the second etc... so the total area is

$$A = L^2\left(1 + \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \cdots\right) = L^2\frac{1}{1-\frac{1}{2}} = 2L^2.$$