

This is a warm-up exam. You should first attempt to solve as much of it as possible on your own. You may discuss it with other students taking Math 221 but you cannot ask a TA or a student more advanced in Calculus to solve it for you. If you cannot solve a problem, try to identify what is preventing you from moving forward. TA's will answer general questions, or questions on suggested problems.

1. [50 pts] Calculate the following expressions. Show your work but NO PARTIAL CREDIT. Full credit only for correct answer with correct derivation.

(1) $\lim_{a \rightarrow \infty} \int_0^a \frac{dx}{1+x^2}$

(2) $\lim_{\theta \rightarrow 0^+} \sqrt{\theta} \ln \theta$

(3) $\int_{-3}^2 \frac{dx}{\sqrt{17-x^2}}$

(4) $\int_2^1 \frac{\ln x}{x} dx$

(5) $\frac{dx^x}{dx}$

(6) $\arccos(\cos(7\pi/4))$

(7) $\cos(\arcsin 3)$

(8) $\sin(\arccos(1/3))$

(9) $\ln(x^3 e^{-x^2})$

(10) $\lim_{x \rightarrow +\infty} \frac{x^a}{a^x}, \quad \forall a > 0$

2. [10pts] Calculate (a) the area between the curves $y^2 = 4ax$ and $y^2 = 8ax - 4a^2$, (b) the volume generated by the rotation of that area about the $x = 2a$ axis.

3. [10pts] A bowl in the shape of a paraboloid (i.e. the surface generated by the rotation of $y = x^2$ about the y -axis) is filled with water at the constant rate of Q m³/sec. How fast is the height of water in the bowl rising?

4. [10pts] A radioactive substance disintegrates at a rate proportional to the amount present. If the rate constant is 1 percent per day, how long will it take for the amount to have reduced by half?

5. [10pts] Suppose you borrow A_0 euros at the rate of r (%/year) and interest is compounded continuously. (a) What is the effective *Annual percentage rate*? (b) If you pay back the money *continuously* at the constant rate of p (euros/year), what is the differential equation that determines the amount of money owed at time t ? (c) Solve that equation.

6. [10 pts] Sketch $y = x^{1/17}$ and $y = \ln x$ on the same plot. Calculate (a) $\lim_{x \rightarrow 0^+} x^{1/17} \ln x$, (b) $\lim_{x \rightarrow +\infty} x^{-1/17} \ln x$. Find a number $M > 0$ such that $x^{1/17} > \ln x$ for all $x > M$.