FW Math 221

Exam warm-up 2 Due Thursday Nov 8, 2001

This is a warm-up for next week's exam. You should first attempt to solve as much of it as possible on your own. You may discuss it with other students taking Math 221 but you cannot ask a TA or a student more advanced in Calculus to solve it for you. If you cannot solve a problem, try to identify what is preventing you from moving forward. TA's will answer general questions, or questions on suggested problems.

1. [10 pts] Evaluate the following limits, showing the main steps in your reasoning.

$$\lim_{x \to 0} \frac{\sin x - x}{\sin x^3} \tag{1}$$

$$\lim_{\theta \to 0} \frac{\tan \theta - \theta}{\cos \theta^3} \tag{2}$$

2. [20pts] Sketch the curves (a) $y = x^2/(1-x^3)$ and (b) $y = 1/x - 1/\sin x$. Identify the exact (not numerical approximations) extrema and inflection points in case (a).

3. [20pts] Calculate the following

$$\int x^2 \cos(x^3 + 1) dx \tag{3}$$

$$\int \frac{d\theta}{\cos^2 \theta} \tag{4}$$

$$\int x^2 \cos x dx \tag{5}$$

$$\int \frac{u}{\sqrt{1-4u^2}} du \tag{6}$$

4. [10pts] The volume of a cylindrical can of soda is 355 cubic centimeters. The can is made of aluminum of fixed thickness. Find the dimensions of the can that requires the least amount of aluminum.

5. [10pts] A spotlight on the ground shines on a wall 12m away. A player from the UW women's volleyball team who is 2m tall is walking toward the wall at 8/5 m/sec. How fast is her shadow on the wall changing when she is 4 m from the wall?

- 6. [10pts] How many zeros does the function $f(x) = (x^2 2x + 2)/(x 1)$ have between 0 and 2?
- 7. [10pts] Solve $dy/dx = 2xy^2$ with y(1) = C. Sketch y(x).
- 8. [10pts] Show that

$$na^{n-1} \le \frac{a^n - b^n}{a - b} \le nb^{n-1}$$

for n = 1, 2, 3, ... whenever $b \ge a > 0$.