

This is a warm-up for next week's exam. You should first attempt to solve as much of it as possible on your own. You may discuss it with other students taking Math 221 but you cannot ask a TA or a student more advanced in Calculus to solve it for you. If you cannot solve a problem, try to identify what is preventing you from moving forward. TA's will answer general questions, or questions on suggested problems.

1. [20 pts] Evaluate the following limits, showing the main steps in your reasoning.

$$\lim_{x \rightarrow -a} \frac{x^2 + a^2}{x^3 - a^3} \quad (a > 0). \quad (1)$$

$$\lim_{\epsilon \rightarrow 0} \frac{\sqrt{4 + \epsilon^2} - 2}{\epsilon^2} \quad (2)$$

$$\lim_{t \rightarrow +\infty} \frac{\sqrt{5 + t^2} - \sqrt{5}}{t} \quad (3)$$

$$\lim_{s \rightarrow 1} \frac{s^2 - 4s + 3}{s^2 + 2s - 3} \quad (4)$$

In relation with limit (3), sketch the curves $u = t$ and $u = \sqrt{5 + t^2} - \sqrt{5}$ for $t \geq 0$. The sketch should illustrate the result you obtained for the limit (3). Specify the *exact* values of t at which the curves intersect, if they do ($\sqrt{2}$ is exact but 1.414 is not).

2. [10pts] What is the range of x such that $\frac{1}{(x+4)^2} > 100$?

3. [20pts] Evaluate dy/dx for each of the following

$$y = \frac{\sqrt{2-x}}{3} \quad (5)$$

$$y = \frac{x}{x^2 - 1} \quad (6)$$

$$y = (x + x^{-1})^2 \quad (7)$$

$$y = b + \sqrt{R^2 - (x - a)^2} \quad (8)$$

4. [10pts] Two variables, s and t , are related by the equation $t = s^3 - s^2 + s$. Find an integer value of s such that $t = 21$. What is the rate of change of s at $t = 21$?

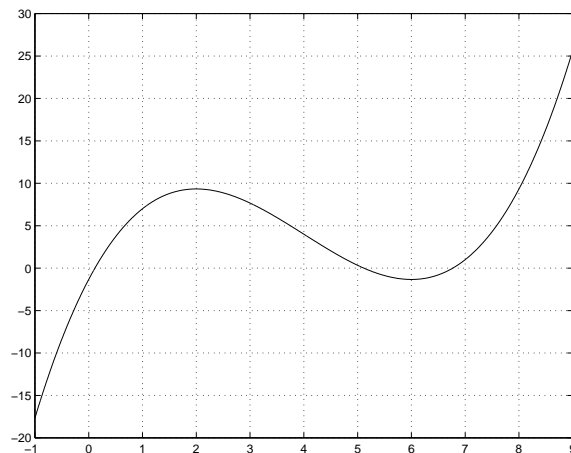
5. [10pts] Consider the function

$$f(x) = \begin{cases} x^2 + bx + c, & \text{for } x < 0 \\ x + 2, & \text{for } x \geq 0 \end{cases}$$

Find the constants b and c such that $f(x)$ and $f'(x)$ are continuous everywhere then sketch $f(x)$.

6. [10pts] The equation $(x^2 + y^2)(x^2 + y^2 + x) = 4xy^2$ implicitly defines a famous curve. What is its slope at $x = 1/2$, $y = \sqrt{3}/2$?

7. [10pts] Sketch the graph of $f'(x)$ where $f(x)$ has the following graph



8. [10pts] Find $f'(0)$, the derivative at $x = 0$, of the function

$$f(x) = \begin{cases} x^2 \sin \frac{1}{x}, & \text{for } x \neq 0 \\ 0 & \text{for } x = 0 \end{cases}$$

[Hint: we have not covered derivatives of trig functions yet but you do not need them here. You need the fundamental definition of the derivative and you can *squeeze* the answer. You may need a sandwich.]