FW EP548 **Take Home Exam 2 due May 6, 2003**

This is a Take home exam. You must work on it alone. You can use the Greenberg book as well as your notes but no other material.

1. Solve $x' = t^2 - x^2$, with $x(0) = 0$ using Euler's method. Can you find the asymptotic behavior as $t \to \infty$? (using hints from your numerical results and asymptotics).

2. Derive a family of Runge-Kutta schemes of order 3 (RK3). Do not hand in scratch work! write up your solution cleanly and concisely.

3. Derive a one-sided (forward) 2nd order finite difference formula for d/dt. Apply the resulting scheme to $x' = -x$ with $x(0) = 1$. Discuss and explain your results.

4. Solve $x'' - \mu(1 - x^2)x' + x = 0$ with $x(0) = 1$, $x'(0) = 0$, using your own numerical code and whatever numerical scheme you prefer, for μ=0.01, 0.1, 1, 10, 100, 1000.

5. Use the method of multiple scales to determine the solution of $x'' - \epsilon(1 - x^2)x' + x = 0$ with $x(0) = b$, $x'(0) = 0$ for small ϵ. Your approximation should be uniformly valid in time up to and including order ϵ.