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On the Origin of Streaks in Turbulent Shear Flows

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Abstract

It is shown that the ideas of *selective amplification* and *direct resonance*, based on linear theory, do not provide a selection mechanism for the well-defined streak spacing of about 100 wall units (referred to as 100^+ hereafter) observed in wall-bounded turbulent shear flows. For the direct resonance theory (Benney & Gustavsson, 1981; Jang et al., 1986), it is shown that the streaks are created by the nonlinear self-interaction of the vertical velocity rather than of the *directly forced* vertical vorticity. It is then proposed that the selection mechanism must be inherently nonlinear and correspond to a *self-sustaining* process. The streak formation is only one stage of the complete mechanism and cannot be isolated from the rest of the process. The 100^+ value should be considered as a critical Reynolds number for that self-sustaining mechanism. For the case of plane Poiseuille flow the 100^+ criterion corresponds to a critical Reynolds number of 1250, based on the centerline velocity and the channel half-width, which is close to the usually quoted value of about 1000. In plane Couette flow, it corresponds to a critical Reynolds number of 625, based on the half velocity difference and the half-width.

1. Introduction

An intriguing feature of wall-bounded turbulent flows is the existence of bands of low- and high-speed fluid, elongated in the streamwise direction, and with a very consistent spanwise spacing of about one hundred wall units, i.e. $100\nu/u_*$, where $u_* = (\nu dU/dy|_w)^{1/2}$ is the friction velocity. The streaks are known to initiate a localized instability that leads to a "bursting process," during which most of the turbulence production takes place (Kline *et al.*, 1967; Kim *et al.*, 1971). Large momentum is exchanged between the wall and the outer fluid during the bursting process, thus sustaining the turbulent flow.

It is now fairly well accepted that the mechanism for streak generation is a rapid one, meaning that the linear distortion of fluctuations by the mean shear strongly dominates the nonlinear effects (Lee *et al.*, 1990). The physical process is a redistribution of downstream momentum by cross-stream motions (downstream rolls) which are decoupled from the mean flow. The important question remaining is to determine what imposes the characteristic streak spacing. A natural suggestion is that the linear mechanism itself leads to a favored scale (selective amplification). The inner-layer of a typical turbulent boundary layer being about 50 wall units thick, the scale of the largest downstream roll would be also around 50^+ and these would induce streaks with a spanwise wavelength around 100^+. The flaw in that reasoning is that it rests on an intuitive, yet false, assumption that rolls are necessarily circular while in fact they can be elliptical. Our analysis indicates that while the linear mechanism provides a scale selection, it is much too weak to be significant and does not correspond to the value of 100^+. The weak scale selection shows up in the vertical vorticity spectra but not in the downstream velocity spectra. In a turbulent boundary layer a peak appears in both spectra.

In the direct resonance theory (Gustavsson 1981; Benney and Gustavsson 1981), oblique vertical vorticity modes are forced by the vertical velocity. The physical process is entirely similar to that for the streak generation mentioned above and consists of a redistribution of momentum in oblique planes by oblique rolls. Because of the obliqueness, the two types of fluctuations (oblique rolls and oblique streaks, or in mathematical terms vertical velocity and vorticity) are, in general, influenced by the mean flow in two different ways. The small vertical velocity fluctuations are governed by the Orr-Sommerfeld equation, while the vertical vorticity fluctuations obey an advection-diffusion equation. The argument is that the forcing of the vertical vorticity by the velocity should be most effective when the eigenvalue of the velocity mode is identical to an eigenvalue of the homogeneous vorticity equation (a so-called direct resonance). The resonance condition is only satisfied for well-defined streamwise (x) and spanwise (z) scales, thus providing a scale selection. If the "resonance" condition is met, the vorticity is expected to reach "large" values. The nonlinear self-interaction of the "large" vertical vorticity then gives rise to downstream rolls (downstream vertical velocity) as shown by Jang, Benney and Gran (1986), and these rolls lead to streaks as explained in the previous paragraph. Unfortunately, our analysis does not support this approach either. The direct resonance criterion does not necessarily select the most amplified vorticity fluctuations. This is because the "resonance" always occurs for damped modes. So the maximum vorticity amplitude that can be obtained is a function of the damping rate, and there are non-resonant modes with a lower damping rate which can reach larger amplitudes. In fact, the largest vorticity amplitudes are obtained for downstream modes, as reported recently by other authors also (Gustavsson 1991; Henningsson 1990), but, as discussed in the previous paragraph, the downstream fluctuations do not yield a significant scale selection. So this approach can not explain the streak spacing either. In any case, our analysis also indicates that the generation of downstream rolls is a result of the nonlinear interaction of the oblique vertical velocities and not of the vorticities, thus bypassing completely the "resonant" amplification of the oblique streaks.

The final part of this paper presents an introduction to ongoing research aimed at providing an understanding of the scale selection and the mechanisms taking place in the near-wall region of turbulent boundary layers. We propose that the streak spacing should be considered as a critical Reynolds number for a self-sustaining nonlinear process, of which streak formation is one of the elements. The nonlinear process would not *persist* for scales lower than about 100^+. This is the case, of course, if the largest scale allowed in the domain is smaller than 100^+. Therefore a link is established between the 100^+ characteristic streak spacing and the critical Reynolds numbers, above which turbulence can be maintained. The 100^+ criterion has the potential to be a more universal value for shear flows than other measures of the critical Reynolds number. It translates into a value of 1250 based on the channel half-width and the centerline velocity in plane Poiseuille flow, and 625 based on the half width and half velocity difference in plane Couette flow (or 1000 and 400 respectively, if the minimum streak spacing is 80^+). The self-sustaining nonlinear process in question would consist of the following elements. First, streaks are created by downstream rolls, the streaks then break down due to an instability of inflexional type initiated by the spanwise inflexions (the vertical shear might significantly influence the nature of that instability). The streak instability leads to the formation of vortices which reinforce the original downstream rolls, and the process repeats itself provided the scale is larger than 100^+; otherwise it loses intensity and eventually decays. The complete process is expected to be itself unstable, or at least "broadband", so that the flow appears disordered if the largest admissible scale is sufficiently bigger than 100^+. In order to capture the process in its simplest form it is necessary to experiment in a domain whose largest scale is close from 100^+.

The idea of selective amplication by the linear nechanism is examined in section 2. The direct resonance theory of Jang et al. (1986) is reexamined in section 3. The self-sustaining mechanism for marginal turbulent flows is presented in section 4, followed by a short summary in section 5.

In this paper, x, y, and z denote streamwise, normal (to the wall), and spanwise directions, respectively, while u, v, and w denote corresponding velocities, respectively.

2. Linear Analysis and Selective Amplification

A number of papers (e.g. Lee *et al.*, 1990) show that the mechanism for streak generation is linear. The argument is that in the near wall-region the time scale for the mean $(dU/dy)^{-1}$ is much shorter than the time scale for the nonlinear effects, measured by q^2/ϵ, where q is a turbulent velocity scale and ϵ is the dissipation rate. The evolution is then dictated by linear equations and streaks are created from the redistribution of the downstream momentum by vertical and spanwise motions. The mechanism is a simple advection and is most efficient when the fluctuating fields are elongated downstream. The question here is to examine whether the linear mechanism favors spanwise scales of about 100^+. The mathematical description of the mechanism is briefly stated in the next few paragraphs.

The governing equations for the fluctuating field, obtained by eliminating pressure and the continuity constraint, are:

$$(\frac{\partial}{\partial t} + U\frac{\partial}{\partial x} - \frac{1}{R}\nabla^2)\nabla^2 v - \frac{d^2U}{dy^2}\frac{\partial}{\partial x}v = NL_v \tag{1}$$

$$(\frac{\partial}{\partial t} + U\frac{\partial}{\partial x} - \frac{1}{R}\nabla^2)\,\eta + \frac{dU}{dy}\frac{\partial}{\partial z}v = NL_\eta \tag{2}$$

where v and η denote respectively the y-component of velocity and vorticity, and U is the mean velocity. The right-hand sides, NL_v, NL_η, represent nonlinear terms. Flows in a channel will be considered in this paper (plane Poiseuille or plane Couette flow) with the boundary conditions $v = \partial v/\partial y = \eta = 0$ at the walls, located at $y = \pm 1$.

In the linear case (NL_v=0), the equation for v is homogeneous and admits eigensolutions of the form:

$$v = \hat{v}(y)e^{i(\alpha x + \beta z - \omega t)} \tag{3}$$

where $\hat{v}(y)$ satisfies the *Orr-Sommerfeld* equation. In general, for a turbulent mean profile, all of these eigensolutions are decaying. The η-equation on the other hand, is non-homogeneous for v fluctuations with a spanwise variation. When forced by an eigenmode of the v equation, the linear response of the vertical vorticity has the form:

$$\eta = \hat{\eta}(y,t)e^{i(\alpha x + \beta z)} \tag{4}$$

with $\hat{\eta}(y,t)$ given by:

$$\hat{\eta}(y,t) = \beta \sum_n \lambda_n \,\frac{e^{-i\omega t} - e^{-i\mu_n t}}{\omega - \mu_n}\, \eta_n(y) \tag{5}$$

Where

$$\lambda_n = \frac{\int U'\,\hat{v}\;\eta_n\;dy}{\int \eta_n \eta_n dy} \tag{6}$$

and $\eta_n(y)$, μ_n represent the eigenmodes and eigenvalues of the homogeneous η-equation. Note that the vertical vorticity response corresponds to streaks, as opposed to vortices. The physical mechanism is a redistribution of momentum or vorticity *tilting*, not *stretching*.

When the OS eigenvalue ω is close to the Squire eigenvalue μ_n, the n-th coefficient will behave as $t\exp(-i\mu_n t)$. This corresponds to an algebraic growth followed by exponential decay, as μ_n corresponds to a viscously decaying mode. For a significant algebraic growth to occur the real parts of ω and μ must be sufficiently close. Otherwise the modes will decorrelate. Furthermore, the viscous damping must be small. Thus one expects and verifies numerically that the largest responses occur for downstream modes ($\alpha = 0$) for which the real parts of the eigenvalues vanish (downstream modes do not feel the mean flow) and the decay rate is inversely proportional to the Reynolds number.

The possibility of a scale selection by the linear mechanism was investigated numerically, by introducing a downstream OS mode, normalized such that the maximum vertical velocity was unity, onto a turbulent mean profile with $R = 180$, based on the friction velocity and the channel half-width. The mean velocity profile was chosen as a Reynolds-Tiederman profile, defined by

$$\frac{dU}{dy} = -\frac{R\,y}{(1+\nu_t)}$$

$$\nu_t = \frac{1}{2}\{1 + [\frac{1}{3}KR(1-y^2)(1+2y^2)$$
$$(1 - \exp(-(1-|y|)R/A))]^2\}^{1/2} - \frac{1}{2}$$

where K and A were chosen respectively as 0.525 and 37.

The forced responses of the vertical vorticity are shown in Fig.1. It can be seen that there is a peak around $\lambda_z^+ = 35$, but it is too weak to represent a significant selection. The streamwise *velocity* responses is obtained by multiplying the vorticity by the wavelength and the largest response corresponds to the largest wavelength. This does not match the experimental observations, which show a scale selection both in the velocity and vorticity spectra. In any case, the "peak" does not correspond to the typical value for the streak spacing, which is between 80^+ and 100^+. We must conclude that the linear mechanism does not provide a scale selection.

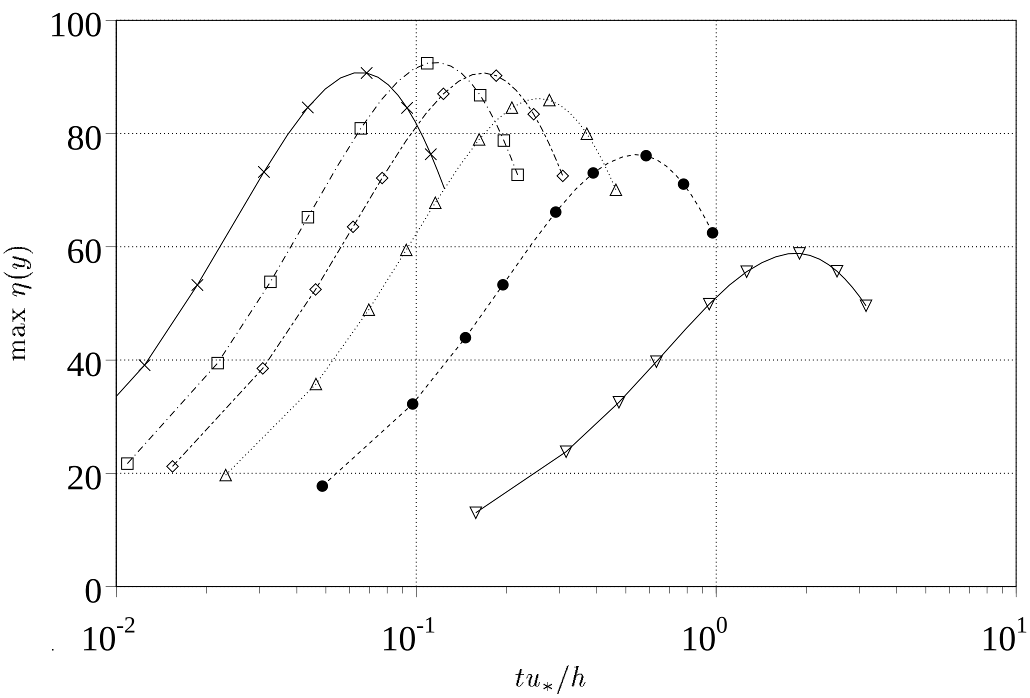

Fig. 1. Maximum pointwise vertical vorticity response to a single downstream even OS mode. ▽ , $\lambda_z^+ = 188$; ● , $\lambda_z^+ = 94$; △ , $\lambda_z^+ = 63$; ◇ , $\lambda_z^+ = 47$; □ , $\lambda_z^+ = 38$; × , $\lambda_z^+ = 27$. *Linear theory does not lead to sharp scale selection.*

3. Direct Resonance Theory

In the direct resonance scenario, streaks originate from a three-step process (Benney and Gustavsson, 1981; Jang *et al.*, 1986) The first step is linear and consists of the resonant forcing of the vertical vorticity by the velocity, exactly as in the previous section, but focuses on oblique disturbances, for which the nonlinear effects can be less trivial. The second step is the nonlinear interaction of the vorticity with its mirror image across a vertical downstream plane. This would create downstream vortices which, finally, give rise to the streaks. That sequence of interactions is illustrated by the following diagram,

$$\begin{aligned} \epsilon\, v(\alpha, \pm\beta) &\longrightarrow \epsilon t\, \eta(\alpha, \pm\beta) \\ \epsilon^2 t^2\, \eta\eta^* &\longrightarrow \epsilon^2 t^3\, v(0, 2\beta) \\ \epsilon^2 t^3\, v(0, 2\beta) &\longrightarrow \epsilon^2 t^4\, \eta(0, 2\beta) \end{aligned}$$

One can imagine that this would then be completed by the non-linear interaction of the downstream streaks ($\eta(0, 2\beta)$) with the oblique ones ($\eta(\alpha, \pm\beta)$) to generate $v(\alpha, \pm\beta)$ which then would recreate the oblique streaks and so on,

$$\epsilon^3 t^5\, \eta(\alpha, \pm\beta)\eta(0, 2\beta) \longrightarrow \epsilon^3 t^6 v(\alpha, \pm\beta) \longrightarrow \epsilon^3 t^7 \eta(\alpha, \pm\beta)$$

The expansions for $v(\alpha, \pm\beta), \eta(\alpha, \pm\beta)$ would be of the form

$$\begin{aligned} v(\alpha, \pm\beta) =& \epsilon\, (1 + \epsilon^2 t^6 + \ldots)\, e^{i(\alpha x \pm \beta z - \omega t)} \\ \eta(\alpha, \pm\beta) =& \epsilon t\, (1 + \epsilon^2 t^6 + \ldots)\, e^{i(\alpha x \pm \beta z - \omega t)} \end{aligned}$$

These expansions suggest that the relevant time scale for the non-linear effects could be as fast as $\epsilon^{-1/3}$. This is even faster than the $\epsilon^{-1/2}$ time scale assumed by Benney and Gustavsson (1981) in their non-linear theory based on direct resonances. The reason for this difference is that Benney and Gustavsson overlooked the possibility of secular terms arising from streamwise independent modes such as $v(0, 2\beta)$.

The creation of downstream rolls $v(0, 2\beta)$ from the non-linear interactions of oblique streaks $\eta(\alpha, \pm\beta)$ is the subject of the work of Jang *et al.* (1986). Using a turbulent boundary layer profile, Jang *et al.* found that an OS eigenvalue coincided with a "Squire" eigenvalue (a *direct resonance*) for the horizontal wavenumbers, $\alpha^+ = 0.0093$ and $\beta^+ = 0.035$. The common eigenvalue was equal to $\omega^+ = 0.090 - i\, 0.037$. They showed that the interaction of that mode with its spanwise reflexion induced streamwise vortices with a spanwise wavelength around $\lambda^+ = 2\pi/\beta^+ \simeq 90^+$.

One problem with the direct resonance concept is that it must assume not only that the eigenvalues of the two linear homogeneous operators are close, but also that the damping rates are small. Otherwise, as discussed above, the algebraic growth will be quickly shut off by the exponential viscous decay. Another problem comes from multiple resonances or near-resonances. There is no a priori criterion for which one should prevail.

The possibility of the direct resonance route to streaks has been explored in the turbulent channel flow (Kim *et al.*, 1987). The Reynolds number is about 180, based on the channel half-width and the friction velocity. A direct resonance is found for

$$\alpha^+ \simeq 0.005 \quad \beta^+ \simeq 0.039$$

with the corresponding eigenvalues

$$\begin{aligned} \omega^+_{14} =& 0.07871485 - i\, 0.02168581 \\ \mu^+_{15} =& 0.07871461 - i\, 0.02168558 \end{aligned}$$

These values are different but close to the values reported above for the turbulent boundary layer. The nonlinear interaction of the pair of modes $(\alpha, \pm\beta)$ leads to streaks with a spanwise spacing of about 80^+. The vertical vorticity responses are displayed in figure 2. The initial conditions were such that the maximum v amplitude was 0.1 with no vertical vorticity. As indicated by the subscripts, this direct resonance occurs at the 14th OS mode and the 15th Squire mode, where the modes are ordered according to their decay rate. Although this result looks encouraging, the picture is not as clear when one analyzes other modes.

For instance the linear response obtained from the 13th OS mode for slightly different wavenumbers is very similar to the direct resonance mode, but the nonlinear response is even larger (Fig.2). The response to the 19th OS mode for $(\alpha, \beta) = (1.6, 12)$ revealed rather confusing results (Fig.3). The linear response is almost four times larger than the direct resonance one, but the nonlinear response is much smaller. One would expect that if the linear response was four times larger than for the direct resonance modes, the nonlinear response should be sixteen times larger than in the direct resonance case.

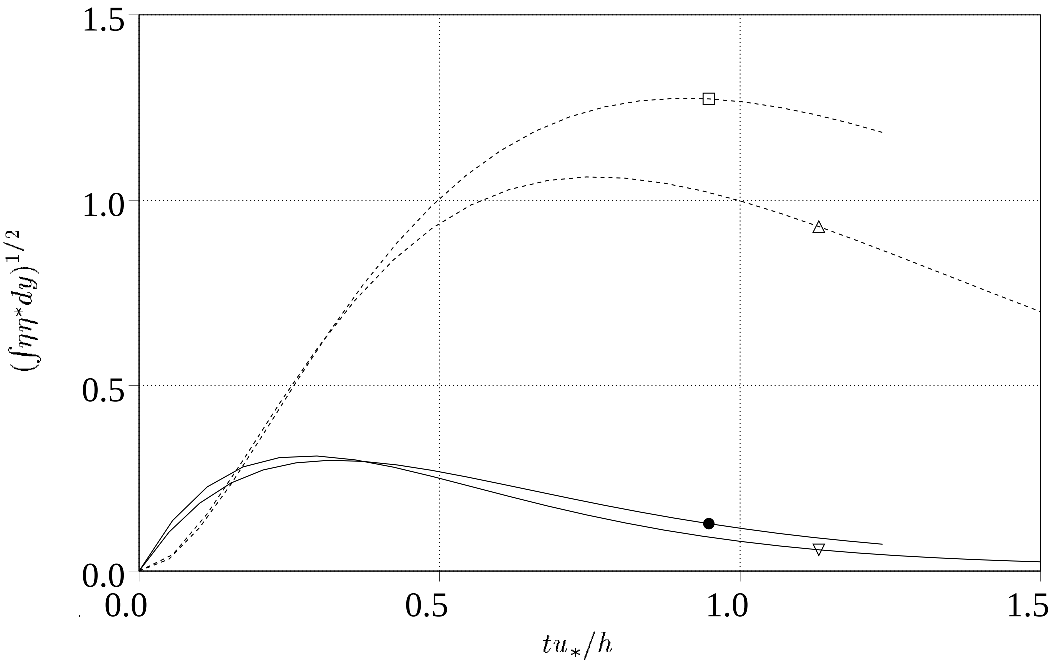

Fig. 2. Linear (solid curves) and non-linear (dashed curves) vertical vorticity response for direct resonance: ▽ , $(\alpha, \beta) = (0.9, 7.07)$ (forced by 14th OS mode); △ , $(0, 14.14)$; ● , $(0.8, 6)$ (forced by 13th OS mode); □ , $(0, 12)$. *Direct resonance does not predict maximum amplification.*

To understand what goes on it is necessary to analyze the nonlinear interactions. The computations and the theory are started with a pair of oblique rolls such as:

$$v = \cos\beta z\,(v_1(y,t)e^{i\alpha x} + v_1^*(y,t)e^{-i\alpha x})$$

The linear processes then introduce a pair of oblique streaks:

$$\eta = \sin\beta z\,(\eta_1(y,t)e^{i\alpha x} + \eta_1^*(y,t)e^{-i\alpha x})$$

The nonlinear effect of primary interest is the generation of streamwise vortices $V(y,t)\cos 2\beta z$. The complete equation for $V(y,t)$ is obtained from (1) with the nonlinear forcing

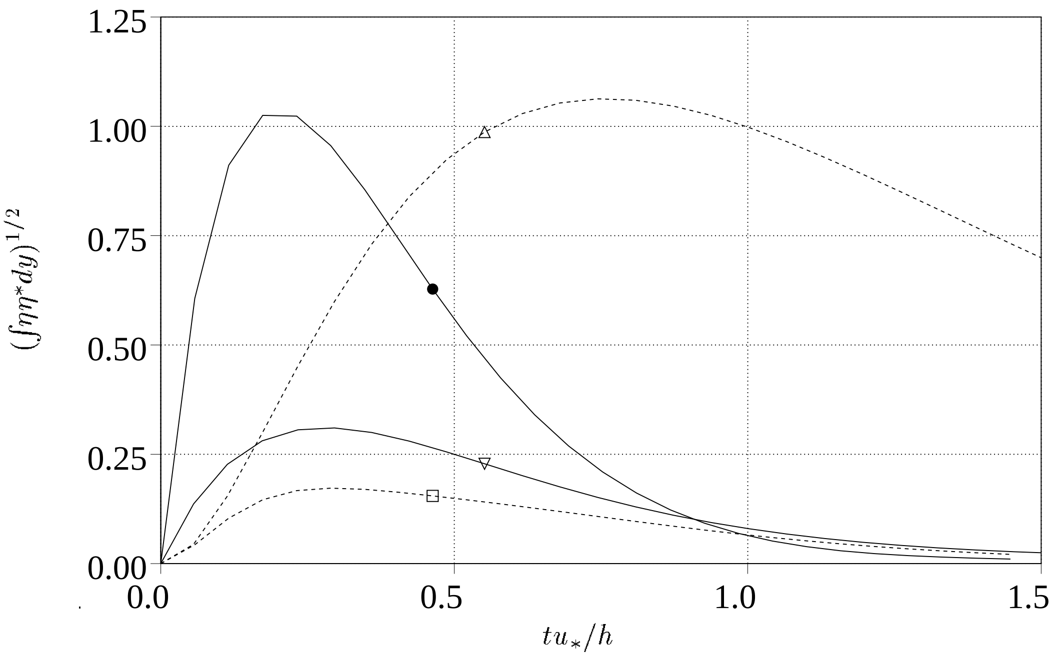

Fig. 3. Linear (solid curves) and nonlinear (dashed curves) vertical vorticity response for direct resonance: ▽ , $(\alpha, \beta) = (0.9, 7.07)$ (forced by 14th OS mode); △ , $(0, 14.14)$; ● , $(1.6, 12)$ (forced by 19th OS mode); □ , $(0, 24)$. *The largest linear amplification does not lead to largest nonlinear response!*

provided by the pair of oblique rolls. After some manipulations (see e.g. Benney, 1961; Lin and Benney, 1962), one finds:

$$[\frac{\partial}{\partial t} - \frac{1}{R}(\frac{\partial^2}{\partial y^2} - 4\beta^2)]\,(\frac{\partial^2}{\partial y^2} - 4\beta^2)\,V =$$

$$\beta(\frac{\partial^2}{\partial y^2} + 4\beta^2)(v_1 w_1^* + v_1^* w_1) + 4\beta^2 \frac{\partial}{\partial y}(v_1 v_1^* + w_1 w_1^*) \qquad (7)$$

where w_1 is given in terms of v_1 and η_1 by

$$w_1 = k^{-2}\,(-\beta \frac{\partial v_1}{\partial y} + i\alpha\eta_1)$$

with $k^2 = \alpha^2 + \beta^2$. The right hand side of (7) can be rewritten as the sum of three forcing terms $F_{vv} + F_{v\eta} + F_{\eta\eta}$, where

$$F_{vv} = 4\frac{\alpha^2\beta^2}{k^2}\frac{\partial}{\partial y}(v_1 v_1^*) + 4\frac{\beta^4}{k^4}\frac{\partial}{\partial y}(\frac{\partial v_1}{\partial y}\frac{\partial v_1^*}{\partial y}) - \frac{\beta^2}{k^2}\frac{\partial^3}{\partial y^3}(v_1 v_1^*)$$

$$F_{v\eta} = \frac{i\alpha\beta}{k^2}[(\frac{\partial^2}{\partial y^2} + 4\beta^2)(\eta_1 v_1^* - \eta_1^* v_1) + \frac{4\beta^2}{k^2}\frac{\partial}{\partial y}(\eta_1^* \frac{\partial}{\partial y} v_1 - \eta_1 \frac{\partial}{\partial y} v_1^*)]$$

$$F_{\eta\eta} = \frac{4\alpha^2\beta^2}{k^4}\frac{\partial}{\partial y}(\eta_1\eta_1^*)$$

In the direct resonance theory, the oblique streaks (η_1) are supposed to be much larger than the rolls so that only their nonlinear interaction, i.e. the term $F_{\eta\eta}$, is considered. The three forcing terms for the direct resonance case at the time when $\eta(\alpha, \pm\beta)$ reaches its maximum amplitude (cf. figure 2) are shown in figure 4. Figure 4 shows that the dominant forcing term is the nonlinear interaction of the oblique rolls (v_1) and not of the "directly forced" oblique streaks (η_1). Thus the streamwise vorticity is created by the nonlinear interaction of the vertical *velocity* of the oblique rolls, not the vertical *vorticity* as proposed in the direct resonance theory. We must conclude that the formation of streamwise vortices, and the streaks they induce, are not directly associated with "direct resonances."

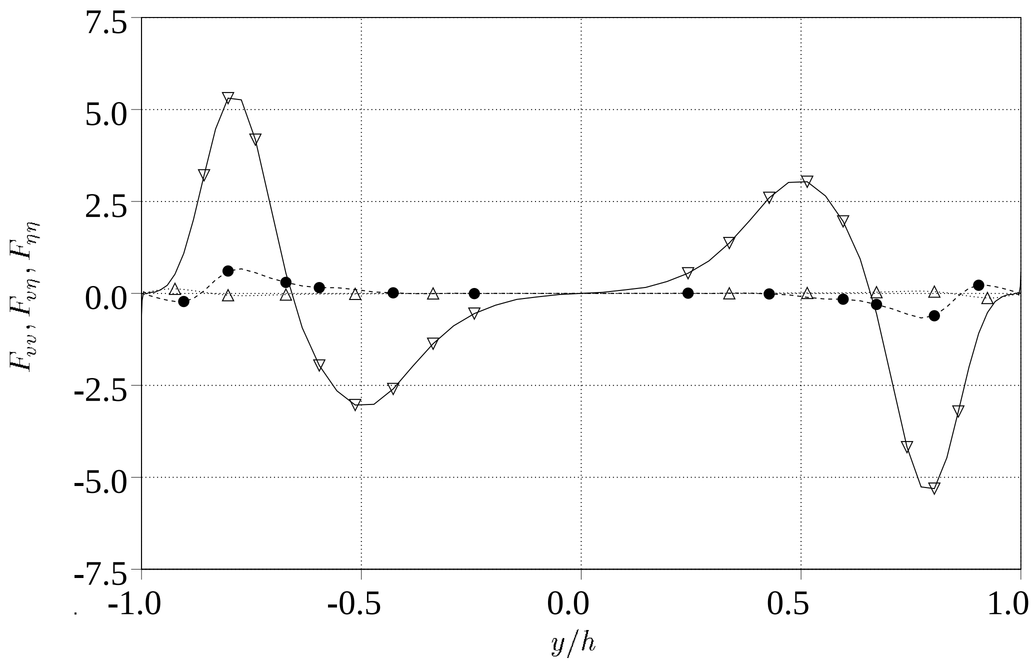

Fig. 4. Forcing terms for the direct resonance mode at time when η is maximum: $\triangledown$, F_{vv}; $\bullet$, $F_{v\eta}$; and $\triangle$, $F_{\eta\eta}$. *Nonlinear effects do not arise from nonlinear interaction of the algebraically amplified disturbances.*

4. Marginal Flow

In view of the failure of the linear mechanism to provide a scale selection, we hypothesize that the selection must come from the original disturbance. In a turbulent flow the disturbance seems to arise from the breakdown of the streaks themselves (the bursting process). Thus the 100^+ selection must come from the complete *self-sustaining* mechanism. Our conjecture is that disturbances with a spacing smaller than about 100^+ cannot be maintained. The 100^+ should then be considered as a *critical Reynolds number*. This conjecture is consistent with the work of Jimenez and Moin (1991). They show that for a channel flow at three different Reynolds numbers, $U_c h/\nu = 5000$, 3000 and 2000, (U_c is the centerline velocity of the equivalent laminar flow, h is the half-height), the flow returns to a laminar state when the spanwise width of the periodic box is reduced below about 100^+.

We observe that for channel width of about 100^+ the channel is very narrow. The characteristic length for the scaling of the disturbance should then be taken as the half width as opposed to the half height. In more general terms, the characteristic length should be taken as the smallest of half the channel height and width. Doing so it turns out that turbulence can not be maintained if $u_* \lambda_z/\nu < 100$, which corresponds to $U_c\lambda_z/(2\nu) < 1250$ ($\lambda_z/2$ is the half-width), irrespective of the value of $U_c h/\nu$. But 1250 is close to the usually quoted value for the critical Reynolds number in channel flow. If one had reduced the height h as opposed to the width λ_z, "turbulence" would have also disappeared when $U_c h/\nu < 1250$ or $u_* h/\nu < 100$.

In order to capture the self-sustaining process in its simplest form, the next step is to reduce both dimensions ($2h$ and λ_z) to their minimum value, so as to eliminate all unnecessary scales. One should be reminded at this point that the streaks are expected to be an essential element of the whole mechanism. The streaks are created from the redistribution of downstream momentum. From the distribution of the mean shear in a channel flow, the simplest self-sustaining non-laminar flow should then consist of a pair of opposite streaks in the spanwise direction and in the direction perpendicular to the walls as well, i.e. one pair of streaks in either half of the channel. In Couette flow, where the mean shear has only one sign, the simplest solution should correspond to only one pair of streaks in the middle of the channel. Thus, the simplest marginal channel flow should have dimensions $u_*\lambda_z/\nu \simeq 100$ and $u_* 2h/\nu \simeq 100$ (corresponding to Re=1250, based on the centerline velocity), while the simplest marginal Couette flow should have $u_*\lambda_z/\nu \simeq 100$ and $u_* 2h/\nu \simeq 50$ (corresponding to Re=625, based on the half velocity difference). According to Smith and Metzler (1983), the smallest value for the streak spacing is rather about 80+, a value which then corresponds to a critical Reynolds number of 400 in Couette flow.

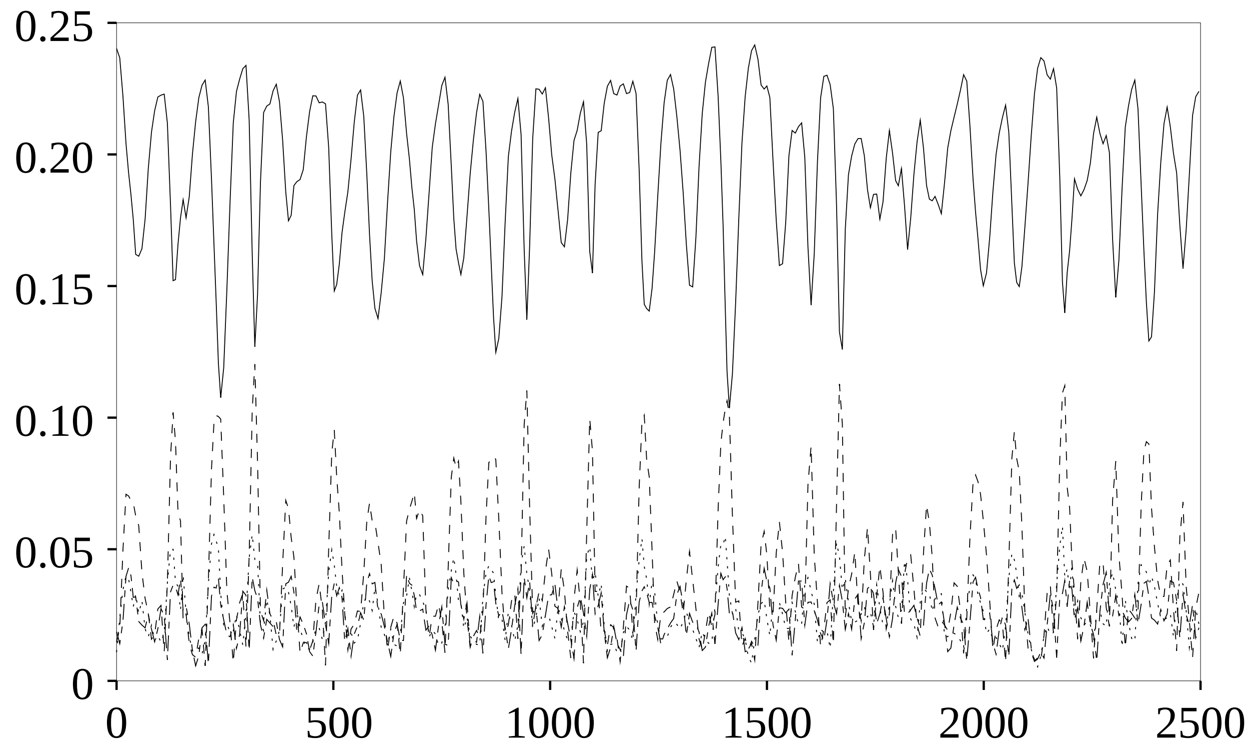

Fig. 5. Time history of the y-averaged energies of various Fourier components in a marginal Couette flow at Re=400. Solid is x-independent Fourier mode, others are streamwise dependent modes.

A number of simulations of both flows were performed, and the results support the above reasoning. Turbulent Couette flow, for instance, could not be maintained at Reynolds numbers of 330 and below (based on the half height and half velocity difference) but was maintained for over 2000 convective time units ($2h/2U_w$) at a Reynolds number $U_w h/\nu = 400$. Thus the non-laminar flow was maintained for over 5 viscous units (h^2/ν), a time scale over which the slowest decaying scales, i.e. $u(y,z) = \cos(\pi y/2h)\cos(2\pi z/\lambda_z)$ with $\lambda_z = 4h$, would decay by a factor $\exp(-2.5\pi^2) \simeq 1.9 \times 10^{-11}$ if they were not sustained. The computed flow fields indicate strong similarity to those obtained in the near-wall region at a higher Reynolds number (e.g. Lee & Kim 1991). The main mechanism appears to be the breakdown of the streaks caused by a spanwise inflectional instability. This is illustrated in Fig.5 which displays a partial time history of the y-averaged energy of the main Fourier components of the flow. The most significant aspect of this plot is the near-periodic behavior on a time-scale of the order $100h/U_w$, which is of the same order as the intermittency cycle observed by Jimenez and Moin (1991). The streaks ($\alpha = 0$) and the other modes ($\alpha \neq 0$) have nearly opposite phases in support of the contention that it is the breakdown of the streaks which energizes these x-dependent modes. The energy of the $\alpha = 0$ modes is entirely dominated by the streamwise component, note then that the ratio of the energies in the streaks to the other modes is of the same order as the ratio of the energies in the streamwise fluctuating velocity to the other components in the near-wall region of a turbulent flow (e.g. Kim *et al.* (1987)). Figure 6 shows contours of streamwise velocity at eight successive times between t=162 and t=262, corresponding to the third cycle on Fig.5.

5. Summary

The streaks, well-defined bands of low- and high-speed fluid present in the near-wall region of turbulent shear flows, are an integral part of the bursting process, by which momentum is exchanged between the wall and the outer fluid. The formation of the streaks is well described by linear theory and corresponds to the rotation of the vertical mean shear by downstream rolls to create a spanwise shear. The linear theory, however, does not provide an explanation for the well-defined scale selection (100^+). The amplitudes of the streaks obtained from the linear equations for a range of scales around the observed 100^+ do not show any significant peak. The direct resonance theory (Jang et al., 1986) is based on the same linear mechanism but for oblique disturbances, whereby oblique rolls create large oblique streaks. A pair of these large oblique streaks then interact nonlinearly to yield downstream rolls, which themselves create downstream streaks. In the direct resonance theory, the scale selection comes from the fact that the creation of oblique streaks by oblique rolls would be more efficient for some particular scales. Unfortunately, this is not found to be the case. In fact the largest responses occur for downstream modes which do not show any significant scale selection as stated above. Furthermore the analysis of the numerical experiments indicates that the downstream rolls are created by the self-interaction of the oblique rolls rather than that of the oblique streaks.

Our conclusion is that the scale selection comes from the complete self-sustaining nonlinear mechanism which consists of the creation, destruction and regeneration of the streaks. The observed characteristic scale of about 100 wall units corresponds to a critical Reynolds number for that process, below which it would not be self-sustaining. This establishes a link between the 100^+ streak spacing and critical Reynolds numbers below which turbulent flows can not be maintained. Numerical investigations support this view. Our goal is to determine the structure of the flow in order to deduce a simple dynamical theory of the whole non-linear process.

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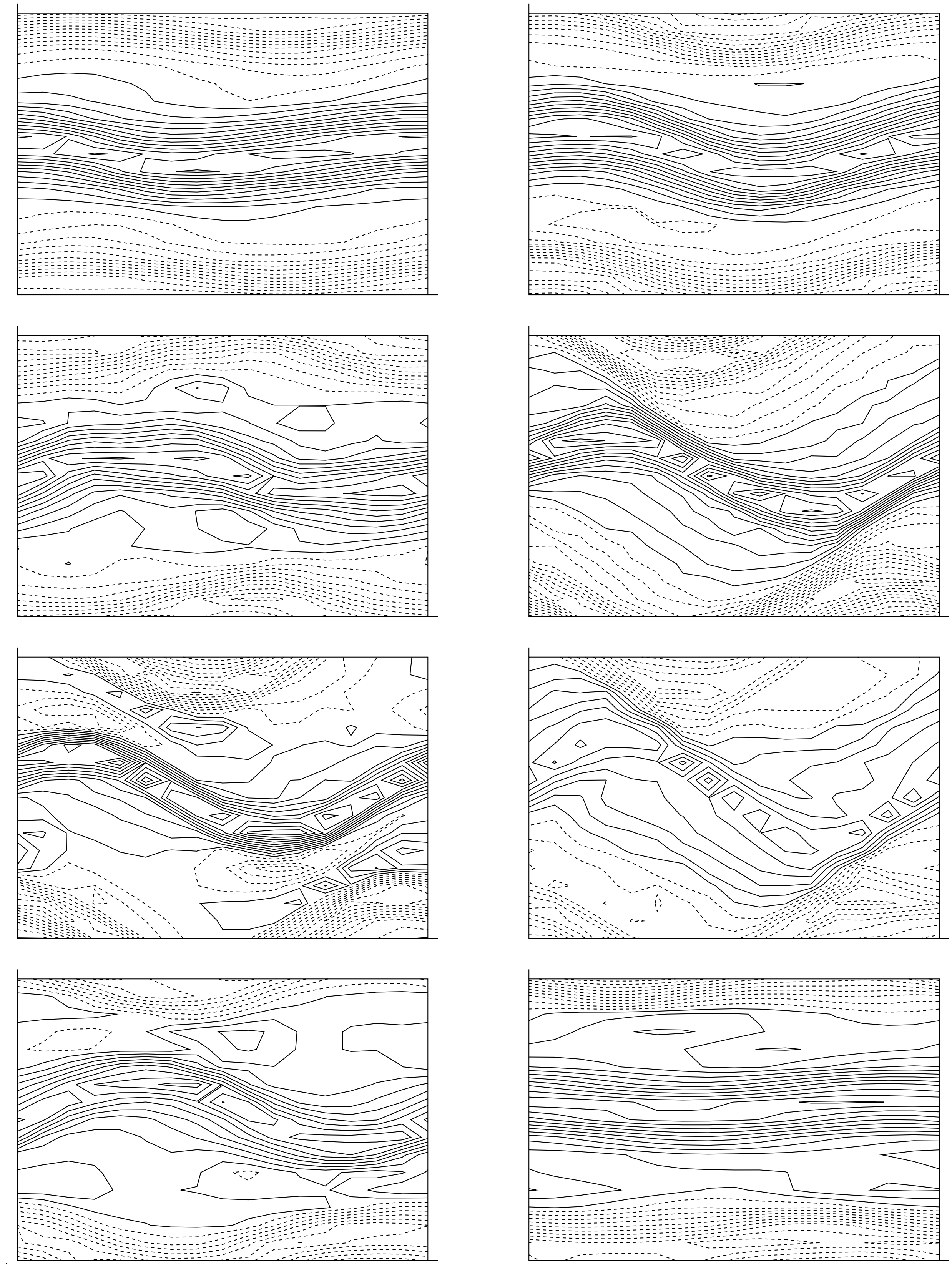

Fig. 6. Contours of streamwise velocity at eight successive times (time is from left to right and from top to bottom) illustrate the near-periodic formation and breakdown of streaks in a marginal plane Couette flow. Contours shown are drawn in the (x, z)-plane in the *middle* of the channel. Top wall moves to the right, bottom one to the left.

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This paper has been rebuilt by FW after publication. Figs. 5 and 6 are not the originals, but are equivalent.