A groupwork day. First, remind them of the theorem from last time.

The solutions of the differential equation

$$ay'' + by' + cy = 0$$

are

- $c_1 e^{r_1 x} + c_2 e^{r_2 x}$, when the polynomial $aC^2 + bC + cC$ has distinct real roots r_1, r_2.
- $c_1 e^{px} \cos(qx) + c_2 e^{px} \sin(qx)$, when the polynomial above has complex roots $p \pm iq$.
- $(c_1 + c_2 x)e^r x$ when the polynomial has only the single root r.

Now–suppose we have an equation like:

$$y'' + 16y = 80$$

This is like a spring in a stiff wind.

How to solve it? Well, I might think of one solution, namely $y = 5$. OK, but what are all the solutions? Well, observe the following. I claim $y = 5 + c_1 \cos(4x) + c_2 \sin(4x)$ is also a solution. For then

$$y'' = -16c_1 \cos(4x) - 16c_2 \sin(4x)$$

and adding it up we get the right answer.

FACT: Suppose y_0 is a solution to

$$ay'' + by' + cy = f(x).$$

Then the set of solutions of the equation above is

$$y_0 + \{\text{solutions of } ay'' + by' + cy = 0.\}$$

Groupwork:

- Find all solutions to the equation
$$y'' + 4y' + 5y = 0.$$
- Find all solutions to the equation
$$y'' + 4y' + 5y = 5x - 1.$$

If by any chance we get through all this, give an example of setting initial conditions.