

## Saturday, November 7

NOTE: Unless otherwise specified, all talks will take place in the ninth floor lounge

### 8:30 - Breakfast

*Ninth floor lounge*

### 9:00 - Galois Groups of Random $p$ -adic Polynomials

*Benjamin Weiss*

The space of fixed degree polynomials with  $p$ -adic coefficients has a natural probability distribution. Each polynomial also has an associated group which is the Galois group of its splitting field. We will discuss the induced distribution on groups, and derive results for the limiting distribution as  $p$  grows. Time permitting, we will discuss a relationship to Serre's mass formulae for extensions of local fields, and prove a complementary theorem to the Chebotarev Density theorem. This work is joint with Chris Hall.

### 9:30 - Counting Cubic Fields and Rings

*Ari Shnidman*

I will present asymptotics for the number of cubic rings of bounded discriminant having a fixed lattice shape. As one consequence, one obtains a generalization of Cohen's theorem on the number of abelian cubic fields of bounded discriminant, namely asymptotics for the number of cubic fields (or orders) having a fixed quadratic resolvent field. This is joint work with Manjul Bhargava.

### 10:00 - Lehmer's Digit Reversal Problem

*Johann Thiel*

Given an integer base  $b$ , and a positive integer  $n$ , let  $\bar{n}$  be the integer obtained by reversing the digits of  $n$  in base  $b$ , and let  $f(n) = n + \bar{n}$ . Lehmer's digit reversal problem asks whether, for any positive integer  $n$ , the sequence  $f(n), f(f(n)), f(f(f(n))), \dots$  eventually contains a palindrome in base  $b$ . We survey known results on this problem, discuss probabilistic aspects, and report on the result of large-scale numerical investigations.

### 10:30 - New Distance Bounds for Codes on Algebraic Curves

*Radoslav Kirov*

Using a construction due to Goppa, algebraic curves can be used to produce error-correcting codes. A method of Feng-Rao and its generalization - the order bound - have been successful in obtaining bounds for the distances of such codes. We translate those methods to a geometric setting and generalize them. We define certain subsets of the semigroups of base point free divisors classes, which capture the supports of subsets of words in the code. Our main theorem gives a computational algorithm to get bounds on those subsets using only intrinsic properties of Riemann-Roch spaces. Explicit applications of our method to Hermitian curves are presented.

### 11:00 - Invited Talk: Exceptional Polynomials

*Michael Zieve*

The bulk of Dickson's thesis (1896, Chicago) was devoted to what are now called "exceptional polynomials": those polynomials  $f \in \mathbf{F}_q[x]$  for which the map  $a \mapsto f(a)$  induces a bijection on  $\mathbf{F}_{q^n}$  for infinitely many  $n$ . I will present theorems and conjectures describing all such polynomials. The proofs use Weil's Riemann hypothesis for function fields, Galois theory, and the classification of finite simple groups, and lead to questions and results about automorphism groups of curves.

### 12:00 - Lunch

### 2:00 - Multiplicities of Galois Representations in Cohomology Groups of Shimura Curves

*Chuangxin Cheng*

Multiplicity one result was thought to be a crucial ingredient of the Taylor-Wiles construction as well as other parts of Wiles's proof of Shimura-Taniyama conjecture. Diamond explained how we can get multiplicity one results as a byproduct rather than an ingredient. In this talk, I will show how to use Diamond's idea to

prove multiplicity one results for Galois representations in cohomology groups of Shimura curves.

### **2:30 - Integrality and Non-Vanishing Mod $p$ of the Yoshida Lift**

*Johnson Jia*

Yoshida lift is an instance of theta liftings in the theory of automorphic forms. In this case, the pair of (dual reductive) groups we are considering are the (connected component of the) orthogonal similitude group  $GSO(D)$  of a definite quaternion algebra ( $D$  over  $\mathbf{Q}$ ) and the group  $GSp_4$ . The Yoshida lift takes a pair of automorphic forms on  $D^\times$  to a holomorphic Siegel modular form of degree 2. We plan to give an overview of the lift and outline the strategy for proving integrality (in a suitable sense) as well as non-vanishing modulo  $p$ .

### **3:00 - CM Lifting of Abelian Varieties**

*Xinyun Sun*

We provide an exposition of the recent results by Conrad, Chai, and Oort on CM lifting of abelian varieties. We then consider the question of how much control we have over the ramification of the lifting ring. Some partial results are obtained by an adaptation of Norman's methods for lifting abelian varieties using Dieudonne modules.

### **3:30 - Break**

### **4:00 - Genericity of Representations of $p$ -adic $Sp(2n)$ and Local Langlands Parameters**

*Baiying Liu*

Let  $G$  the  $F$ -rational points of the symplectic group  $Sp_{2n}$ , where  $F$  is a non-archimedean local field of characteristic 0. In 2004, J. Cogdell, H. Kim, I. Piatetski-Shapiro, F. Shahidi constructed local Langlands functorial lifting from irreducible generic representations of  $G$  to irreducible representations of  $GL_{2n+1}(F)$ . In 2008, D. H. Jiang and D. Soudry constructed the descent map from irreducible supercuspidal representations of  $GL_{2n+1}(F)$  to those of  $G$ , showing that the local Langlands functorial lifting from the irreducible supercuspidal generic representations is surjective. In this talk, based on above results, using the same descent method of studying  $SO_{2n+1}$  by D. H. Jiang and D. Soudry, I will show the rest of local Langlands functorial lifting is also surjective, and for any local Langlands parameter  $\phi \in \Phi(G)$ , one can construct one representation  $\sigma$  such that  $\phi$  and  $\sigma$  have the same twisted local factors. As applications, I will show the  $G$ -case of a conjecture of Gross-Prasad and Rallis, that is, a local Langlands parameter  $\phi \in \Phi(G)$  is generic, i.e., the representation attached to  $\phi$  is generic, if and only if the adjoint  $L$ -function of  $\phi$  is holomorphic at  $s = 1$ . I will also show for each Arthur parameter  $\psi$ , and the corresponding local Langlands parameter  $\phi_\psi$ , the representation attached to  $\phi_\psi$  is generic if and only if  $\phi_\psi$  is tempered.

### **4:30 - Counting Rational Points on Homogeneous Varieties**

*Thomas Zamojski*

We will give an asymptotic formula for the number of rational matrices of a fixed rational characteristic polynomial of bounded height, settling a new case of Manin's conjecture. In so doing, we will prove an equidistribution theorem for periodic orbits of a Cartan flow, and explain the connection with counting problems. Note that we are not using the theory of unipotent flows in our study.

### **5:00 - Trilinear Forms and Subconvexity of the Triple Product L-function**

**Van Vleck 901**

*Michael Woodbury*

In studying L-functions, a general technique called convexity can be used to give certain bounds on the values of L-functions in families. While the best expected bounds due from the generalized Riemann hypothesis would have many more applications, even just a slight improvement over the convex bound is often quite useful. Such a bound is called subconvexity. Recently Bernstein and Reznikov proved subconvexity of the triple product L-function in the "eigenvalue aspect" using an integral representation of the L-value provided by work of Watson. I will discuss how subconvexity in the "level aspect" can be obtained using a similar formula by Ichino and recent work of Venkatesh to appear in Annals.

### 5:00 - 2-adic and 3-adic Part of Class Numbers and Properties of Central Values of L-functions

*Matija Kazalicki*

We study the connection between 2-parts and 3-parts of the class numbers  $h(-d)$  and  $h(-3d)$  and ray class groups of  $\mathbb{Q}(\sqrt{d})$  unramified outside 2 (and 3), when  $d$  is prime or the product of two primes. We obtain certain “reflection” theorems, and as an immediate consequence we reproduce the result of Williams on divisibility of  $h(-d)$  by 16 when  $d$  is a prime (and we get a similar result in the case when  $d$  is the product of two primes). We also obtain similar congruences for the central values of  $L$ -functions associated to Ramanujan  $\Delta$ -function, and we relate them to the structure of 2-adic and 3-adic Galois representation attached to the  $\Delta$ -function.

## Sunday, November 8

**NOTE:** All talks will take place in the ninth floor lounge

### 9:00 - Ramanujan Congruences for a Family of Eta Quotients

*Jonah Sinick*

Given a sequence of integers  $c(n)$ , define a Ramanujan congruence for  $c(n)$  to be a congruence of the form  $c(Ln + a) \equiv 0 \pmod{L}$  where  $L$  is prime. Hei-Chi Chan showed that the coefficients of the generating function

$$\prod_{n=1}^{\infty} \frac{1}{(1 - q^n)(1 - q^{2n})}$$

obey a Ramanujan congruence (mod 3) and asked if the coefficients obey other Ramanujan congruences. We answer his question in the negative and indicate how to give a complete characterization of Ramanujan congruences for the coefficients of any member of a class of generating functions similar to the generating function above. The talk will be accessible to nonspecialists.

### 9:30 - Ramanujan Congruences in Partition-Theoretic Functions

*Michael Dewar*

It is well-known that the partition function has the congruence  $p(5n + 4) \equiv 0 \pmod{5}$ . Although there are also simple congruences like this for the primes 7 and 11, Ahlgren and Boylan have proven there are no others. In this talk we describe generalizations of this to other partition-theoretic counting functions and to the coefficients of automorphic forms. Applications include overpartitions, crank differences, and 2-colored  $F$ -partitions.

### 10:00 - Explicit Bounds for the Number of $p$ -core Partitions

*Byungchan Kim*

Let  $p$  be a prime number. The generating function for the number of  $p$ -core partitions of  $n$  is

$$\sum_{n=0}^{\infty} pc_p(n)q^n = \prod_{n=1}^{\infty} \frac{(1 - q^{pn})^p}{1 - q^n}.$$

We use the theory of modular forms, and the circle method of Hardy and Ramanujan to derive explicit bounds on  $pc_p(n)$ . As an application, we show Stanton’s conjecture is true for small  $p$ ’s. This is joint work with J. Rouse.

### 10:30 - Transformation Formulas Associated with Integrals Involving the Riemann $\Xi$ -function

*Atul Dixit*

Page 220 of Ramanujan’s Lost Notebook contains a beautiful transformation formula involving the digamma function which is also associated with an integral involving the Riemann  $\Xi$ -function. Here we discuss some new transformation formulas of this type, of which one generalizes Ramanujan’s transformation formula.

Also included are new extensions of formulas of N.S. Koshliakov, A.P. Guinand and W.L. Ferrar.

### **11:00 - $p$ -adic Hypergeometric Series and Supercongruences**

*Dermot McCarthy*

We discuss recent work in which we generalise Greene's hypergeometric series over finite fields in the  $p$ -adic setting. We provide congruences between these  $p$ -adic hypergeometric series and truncated ordinary hypergeometric series. We also relate a special value of the  $p$ -adic hypergeometric series to the  $p$ -th Fourier coefficient of a modular form, thus resolving an outstanding supercongruence conjecture of Rodriguez-Villegas.

### **11:30 - CM Elliptic Curves and Special Values of $p$ -adic L-functions**

*Hunter Brooks*

A technique of Rubin "twists" the Euler system of elliptic units to build canonical rational points on CM elliptic curves. The height and formal group logarithms of these points compute a special value of the Katz 2-variable  $p$ -adic L-function, defined analytically. We will explain this result and state possible generalizations.

### **12:00 - Spectral Identities and Exact Formulas for Counting Lattice Points in Symmetric Spaces**

*Amy DeCelles*

Using the harmonic analysis of  $SL_3(\mathbb{C})$  we discuss the lattice point counting problem in the symmetric space  $SL_3(\mathbb{C})/SU(3)$ . The simplest analogue perhaps is the Gauss circle problem, counting lattice points in Euclidean space. However, the packing arguments used in Euclidean space fail even in the hyperbolic space  $SL_2(\mathbb{R})/SO(2)$  because the ratio of surface area to volume does not tend towards zero as the radius increases. Instead, we use spectral methods to obtain an identity, and then use residue calculus and Perron-type identities to obtain an exact formula, in a way analogous to Riemann-von Mangoldt's explicit formula relating the zeros of the zeta function to the prime numbers.

### **12:30 - Sobolev Spaces and CM points**

*Nicholas Kirchner*

Application of ideas about Sobolev spaces to spaces of automorphic forms establishes a suitable context for various applications to analytic number theory. For example, non-trivial identities among L-functions and CM-point values follow from non- $L^2$  spectral decompositions of products of Eisenstein series.