Review for midterm

- Exam is at 7 pm in McCosh 50. Tell Dana she needs to set up a make-up exam with Cheryl Singleton.
- Exam will cover material through Section 10.6. Remember that you need to know what's in the book. *You are responsible for it whether we discussed it in class or not.*

First of all, let's sum up what we know about series.
Here are the tests. Note that all series here *have positive terms.*

- nth term test. Tells you: divergence. How: $\lim a_n \neq 0$. When to use it: whenever you can. Warning: cannot tell you a series converges!
- Integral test. Tells you: both convergence and divergence. How: If $a_n = f(n)$, with $f(n)$ a decreasing function, then $\sum_{n=1}^{\infty}$ converges if and only if $\int_1^{\infty} f(x)dx$ converges. When to use it: when a_n looks like a function you can integrate. Warning: make sure the function meets the requirements of the theorem! (decreasing, positive, continuous on whole interval.)
- Ratio test. Tells you: both convergence and divergence. How: if $\lim_{n\to\infty} a_{n+1}/a_n < 1$, series converges; if > 1, series diverges. When to use it: when you have factorials. Warning: if the limit is 1, no information!
- Root test. Tells you: both. How: if $\lim_{n\to\infty} a_n^{1/n} < 1$, series converges; if > 1, series diverges. When to use it: when something (hopefully everything) is raised to the nth power. Warning: if the limit is 1, no information!
- Limit-comparison test. Tells you: both. How: If $\sum b_n$ converges, and $\lim a_n/b_n$ exists, $\sum a_n$ converges. If $\sum b_n$ diverges, and $\lim a_n/b_n$ exists and is not 0, then $\sum a_n$ diverges. When to use it: when the series a_n is "like" a series b_n that you know. When nothing else applies. Warning: We can't show divergence by comparing with a convergent sequence! Or convergence by comparing with a divergent sequence!

The first four methods are easy to apply. The fifth requires some art; we have to choose a b_n. The limit-comparison method is really the "mother method," so to be good at this is really to be good at the subject.

What to compare to?

The idea is that given some yucky series like

$$\sum_{n=0}^{\infty} \frac{6n^4 + 7n^3 + 1}{n^6 + 3n^2 + 5}$$

we want to find a simpler series which is "comparable" to it.

bf Rules of thumb.

- Everything beats bounded functions (constants, sin, etc.)
- Powers of n beat logarithms.
- Higher powers of n beat lower powers.
- Exponentials beat powers of n.
- Factorials beat exponentials.

And then the rule of thumb is to drop everything except the "most powerful" term in any sum. Apologize for the "nature, red in tooth and claw" aspect of all this.

For example, in the above, we'd drop all the lower powers of n and be left with the convergent series

$$\sum_{n=0}^{\infty} \frac{6n^4}{n^6} = \sum_{n=0}^{\infty} \frac{6}{n^2}.$$

And indeed, letting $a_n = (6n^4 + 7n^3 + 1)/(n^6 + 3n^2 + 5), b_n = 6/n^2$ gives $a_n/b_n = (6n^6 + 7n^3 + n^2/6n^6 + 18n^2 + 30)$, and the limit of this exists and equals 1.

Ex: Consider

$$\sum_{n=1}^{\infty} (n^2 + 1)(3^n + 2)/(4^n + n).$$

The rule of thumb says, compare it to

$$\sum b_n = \sum_{n=1}^{\infty} n^2 3^n / 4^n.$$

Show the limit is 1, so that the original series converges if this new series converges. But how do we know this new series converges? We can use the trick we used in class–comparing with a geometric series like $(3.5^n/4^n.)$ Maybe easier is using the ratio test. For here

$$b_{n+1}/b_n = (3/4)(n+1)^2/n^2$$

which converges to 3/4.

Challenge them to do

$$\sum_{n=1}^{\infty} \frac{1}{\log(n+1)^3+1} \frac{1}{n}.$$

Take vote, split in pairs for discussion, take vote again, discuss.