Math 542 Homework 7 - Due Thursday Mar. 14

Assume R is a PID.

(1) Let M be the abelian group determined by generators and relations with relations matrix

$$A = \begin{pmatrix} 11 & 5 & 20 \\ 0 & 5 & 0 \\ 20 & 0 & 10 \end{pmatrix}$$

Find the invariant factors of M and the free rank of M. Find the elementary divisors of $Tor(M)$. What is the order of $Tor(M)$? What is the annilhilator of $Tor(M)$?

(2) (a) Classify all finite abelian groups of order 1960.
(b) How many nonisomorphic abelian groups of order 376194854837643697792123 are there?
(c) If a finite abelian group has elementary divisors $2, 2^3, 2^5, 3, 3^2, 3^4, 3^5, 5, 5^2$ what are the invariant factors of this group?

(3) Suppose that $A \in M_{8\times 8}(\mathbb{C})$ has invariant factors $\lambda(\lambda-1)^2$, $\lambda^3(\lambda-1)^2$. Find a rational canonical form and Jordan canonical form that are similar to A.

(4) Suppose that $A \in M_{n\times n}(\mathbb{C})$. Then A is said to be diagonalizable if A is similar to a diagonal matrix. Prove that A is diagonalizable if and only if

$$\mu_A(\lambda) = (\lambda - \alpha_1)\cdots(\lambda - \alpha_k),$$

where $\alpha_1, \ldots, \alpha_k$ are distinct complex numbers.

(5) Let $G = GL(\mathbb{F}_2) = \{A \in M_{3\times 3}(\mathbb{F}_2) \mid \det A \neq 0\}$ where $\mathbb{F}_2 = \mathbb{Z}/(2) = \{\bar{0}, \bar{1}\}$ is the field of integers mod 2. Find representatives of the conjugacy classes of the group G. In other words, give a list of elements of G so that each element of G is conjugate to exactly one element in the list. (Hint: Use Rational Canonical Form).