Math 542 Homework 11 - Due Thursday May 2

(1) Let $f(\lambda) = \lambda^4 - 4\lambda^2 + 2 \in \mathbb{Q}[\lambda]$, let E be the splitting field of $f(\lambda)$ over $\mathbb{Q}$ and let $G = Gal(E/\mathbb{Q})$.

a) Find E and $[E : \mathbb{Q}]$.
b) Find G as a group of permutations of the roots of $f(\lambda)$.
c) Find all of the subgroups of G. Which of there subgroups are normal in G?
d) For each subgroup of G, find the corresponding subfield of $E/\mathbb{Q}$, given generators for the subfield and indicate whether or not the subfield is a normal extension of $\mathbb{Q}$.

(2) Let $f(\lambda) = \lambda^3 - 5$, let E be the splitting field of $f(\lambda)$ over $\mathbb{Q}$ and let $G = Gal(E/\mathbb{Q})$.

a) Find E and $[E : \mathbb{Q}]$.
b) Find G as a group of permutations of the roots of $f(\lambda)$.
c) Find all of the subgroups of G. Which of there subgroups are normal in G?
d) For each subgroup of G, find the corresponding subfield of $E/\mathbb{Q}$, given generators for the subfield and indicate whether or not the subfield is a normal extension of $\mathbb{Q}$.