MATH 376 HOMEWORK 2
DUE THURSDAY FEB. 4

Section 8.9 2a,b, 3, 10-12
Section 8.14 1a,b, 2a

(1) Show that $f(x,y) = \dfrac{x^2y^2}{x^2+y^2}$ has a limit as $(x,y) \to (0,0)$.

(2) (a) Show that $f(x,y) = (y, 2x)$ is continuous at every point $(a,b) \in \mathbb{R}^2$.
(b) Show more generally that any linear transformation $T : \mathbb{R}^n \to \mathbb{R}^m$ is continuous at all $\vec{a} \in \mathbb{R}^n$.

(3) Let $f, g, h : \mathbb{R}^n \to \mathbb{R}$ and suppose that for $\vec{x} \neq \vec{a} \in \mathbb{R}^n$ we have $h(\vec{x}) \leq f(\vec{x}) \leq g(\vec{x})$. Prove that if $\lim_{\vec{x}\to\vec{a}} h(\vec{x}) = L$ and $\lim_{\vec{x}\to\vec{a}} g(\vec{x}) = L$ then $\lim_{\vec{x}\to\vec{a}} f(\vec{x}) = L$.

(**Hint:** Subtract L from the above inequality and think about what happens when you take the absolute value. There will be two cases.)

(4) Let $g : \mathbb{R}^n \to \mathbb{R}^s$ and $f : \mathbb{R}^s \to \mathbb{R}^m$ then $f \circ g : \mathbb{R}^n \to \mathbb{R}^m$ is the composite function defined by $f \circ g(\vec{a}) = f(g(\vec{a}))$.

Prove using the ϵ, δ definition of a limit that if g is continuous at $\vec{x} = \vec{a}$ and f is continuous at $\vec{y} = g(\vec{a})$ then $f \circ g$ is continuous at $\vec{x} = \vec{a}$. (**Hint:** See picture below)