MATH 376 HOMEWORK 1
DUE THURSDAY JAN. 26

Section 8.3 1 ad, 2 dh 4a, 5, 6ab, 7b, 8

Section 8.5 9

(1) Prove that if $\lim_{\vec{x}\to\vec{a}} f(\vec{x}) = \vec{b}$ then for any scalar $\lambda \in \mathbb{R}$ we have $\lim_{\vec{x}\to\vec{a}} \lambda f(\vec{x}) = \lambda\vec{b}$.

(2) Prove that if $\lim_{\vec{x}\to\vec{a}} \|f(\vec{x})\|^2 \to \|\vec{b}\|^2$ then $\lim_{\vec{x}\to\vec{a}} \|f(\vec{x})\| \to \|\vec{b}\|$.

(3) Show that $f(x,y) = \dfrac{x^3 y}{x^6 + y^2}$ does not have a limit at $(x,y) = (0,0)$.

(Hint: compare what happens when you restrict the domain to $y = x^3$ versus what happens along any line through the origin).

(4) Show (using ϵ, δ) that $f(x,y) = x^2 + y^2 - 2x - 4y + 5$ has limit 1 as $(x,y) \to (1,2)$.