MATH 375 HOMEWORK 1
DUE WEDNESDAY SEPT. 9

(1) Prove by contradiction that there are infinitely many primes.

(2) Prove the following formula by induction:

$$1 + 3 + 5 + \cdots + (2n-1) = n^2$$

(3) Prove the following formula by induction:

$$1^3 + 2^3 + \cdots + (n-1)^3 < n^4/4 < 1^3 + 2^3 + \cdots + n^3$$

(4) Let $P(n)$ denote the statement: $1 + 2 + \cdots + n = \frac{1}{8}(2n+1)^2$.
   - (a) Prove that if $P(k)$ is true for an integer k then $P(k+1)$ is also true.
   - (b) Criticize the statement: "By induction it follows that $P(n)$ is true for all n."
   - (c) Amend $P(k)$ by changing the equality to an inequality that is true for all positive integers n.

(5) The Fibonacci numbers are given by the recursive formula

$$a_0 = 1,\ a_1 = 1 \text{ and } a_{n+1} = a_n + a_{n-1} \text{ for } n \geq 1.$$

Prove that for all $n \geq 1$

$$a_n < \left(\frac{1+\sqrt{5}}{2}\right)^n$$

(6) (Well-ordering principle) Show that every non empty collection of positive integers has a smallest member (i.e. If T is a subset of positive integers then there is some $t_0 \in T$ such that for all $t \in T$ we have that $t_0 \leq t$).

Hint 1: Use contradiction followed by induction.

Hint 2: Let S be the set of all positive integers n such that $n < t$ for all $t \in T$.