Math 831 – Spring 2012

Homework 4

Due: Tuesday, October 16th.

1. For each $n \in \{1, 2, \dots\}$, let $\{X_{n,k} : 1 \le k \le n\}$ be i.i.d. random variables such that

$$0 \le X_{n,k} \le C$$

(same constant C for all n and k). Let

$$S_n = X_{n,1} + X_{n,2} + \cdots + X_{n,n}.$$

Assume that

$$\mathbf{E}S_n \to \infty \quad \text{and } VarS_n \to \infty.$$

Show that $S_n \to \infty$ in probability. The natural way to interpret this statement is that for any $k < \infty$, $P\{S_n \le k\} \to 0$ as $n \to \infty$. *Hint*: Since $\mathbf{E}S_n \to \infty$, try to control the distance of S_n to its mean with Chebyshev's inequality.

2. Exercise 2.3.14 on page 72.

3. Exercise 2.4.2 on page 77.