Math 635 – Spring 2012

Homework 5

Due: March 26, 2012, beginning of the class. Late homework will not be accepted.

1. (Exercise 6.1) Use the Itô isometry to calculate the variances of

$$\int_0^t |B_s|^{1/2} dBs \quad \text{and} \quad \int_0^t (B_s + s)^2 dB_s$$

2. (Exercise 6.2) The integrals

$$I_1 = \int_0^t B_s ds$$
 and $I_2 = \int_0^t B_s^2 ds$

are not stochastic integrals although they are random variables. In these cases we just integrate a (random) continuous function the usual (traditional) way. Find the mean and variance of I_1 and I_2 .

3. Use the definition of the Itô integral to prove that

$$\int_0^t B_s^2 dB_s = \frac{1}{3} B_t^3 - \int_0^t B_s ds.$$