Homework 6

Due: Tuesday, November 20th.

1. Exercise 4.3.

2. Exercise 4.5.

3. Exercise 4.9.

4. Exercises 4.10 and 4.11.

5. Two people are bound together and must move around the vertices of a triangle for an eternity. Call the vertices, labeled in a clockwise ordering, as 1, 2, and 3. One of the people, person 1 say, moves at random times (each with exponential distribution with mean $1/\lambda$) in the clockwise direction, and with each move taking the pair to the next vertex. Person 2, on the other hand, jumps the pair after random times (exponential with mean $1/\mu$) in the counter-clockwise direction. Show that the probability that they are at vertex 1 at a given time $t > 0$, given that they are at vertex one at time $t = 0$, is

$$\frac{1}{3} + \frac{2}{3} \exp\left\{-\frac{3(\lambda + \mu)t}{2}\right\} \cos\left\{\frac{\sqrt{3}(\lambda - \mu)t}{2}\right\}.$$

You may use a computer if you want (I would recommend this). However, just be very clear in detailing what you asked the computer to do for you. You need to set up the problem clearly!

Bonus problem:

6. Consider a continuous time Markov chain with state space $\{1, 2, 3, 4\}$ and generator matrix

$$A = \begin{bmatrix} -3 & 2 & 0 & 1 \\ 0 & -2 & 1/2 & 3/2 \\ 1 & 1 & -4 & 2 \\ 1 & 0 & 0 & -1 \end{bmatrix}.$$

Write a Matlab code that simulates a path of this chain. To do so, use the construction provided in the text and in class (i.e. simulate the embedded chain and holding times sequentially and inependently). Using this code and assuming that $X(0) = 1$, estimate $\mathbb{E}X(3)$ by averaging over 1,000 such paths. Note that you will need to make sure you break your "for" or "while" loop after you see that the time will go beyond $T = 3$, without updating the state for that step.