

**Homework 2****Due: Tuesday, Sept. 25th, 2012.**

1. Exercise 1.8 from text. You should draw graphs to convince the grader.
2. Exercise 1.9, (parts a and b) from text.
3. Exercise 1.17 from text.
4. Exercise 1.19 from text.
5. Exercise 1.27 from text.
6. (a) Show that the product of two finite stochastic matrices is stochastic.  
 (b) Show that for stochastic matrix  $P$ , and *any* row vector  $\pi$ , we have  $\|\pi P\|_1 \leq \|\pi\|_1$ , where  $\|v\|_1 = \sum_i |v_i|$ . Deduce that all eigenvalues,  $\lambda$ , of  $P$  must satisfy  $|\lambda| \leq 1$ .
7. Let  $X_n$  denote a discrete time Markov chain with state space  $S = \{1, 2, 3, 4\}$  and with transition Matrix

$$P = \begin{bmatrix} 1/4 & 0 & 1/5 & 11/20 \\ 0 & 0 & 0 & 1 \\ 1/6 & 1/7 & 0 & 29/42 \\ 1/4 & 1/4 & 1/2 & 0 \end{bmatrix}.$$

- (a) Suppose that  $X_0 = 1$ , and that

$$\begin{aligned} &(U_1, U_2, \dots, U_{10}) \\ &= (0.7943, 0.3112, 0.5285, 0.1656, 0.6020, 0.2630, 0.6541, 0.6892, 0.7482, 0.4505) \end{aligned}$$

is a sequence of 10 independent uniform(0, 1) random variables. Using these random variables (in the order presented above) and the simulation strategy presented in class, what are  $X_n$ ,  $n \in \{0, 1, \dots, 10\}$ ? Note, you are supposed to do this problem by hand.

- (b) Using Matlab, simulate a path of  $X_n$  up to time  $n = 100$  using the simulation strategy presented in class. A helpful sample Matlab code has been provided on the course website. Play around with your script. Try different values of  $n$  and see the behavior of the chain. Turn in a few plots (realizations) of your Markov chain together with your Matlab code.