Practice problems for Midterm 2

Part 1 (2 dimensional case: line integrals, areas, fluxes)

(1) Find the integral
$$\int_\Gamma y^2 dx + x^2 dy,$$
where C is given by equation: $(x/a)^2 + (y/b)^2 = 1$, $a, b > 0$.

(2) Find the integral
$$\int_\Gamma xy d\gamma,$$
where Γ is given by $|x| + |y| = 1$.

(3) Find
$$\int_{(-1,2)}^{(2,3)} xdy + ydx\,.$$

(4) Find
$$\int_{ABC} 2(x^2+y^2)dx + (x+y)^2 dy$$
and ABC is triangle based on $A(1,1), B(2,2), C(1,3)$ with counter-clock wise direction.

(5) Find the area inside the curve given by $r^2 = \cos(2\phi)$ in polar coordinates.

Part 2 (3 dimensional case: Volumes, triple integrals, surfaces, fluxes).

(1) Find the volume of the solid given by intersection of $z^2 \leq x^2 + y^2$ and $x^2 + y^2 + z^2 \leq 1$. Draw the picture.

(2) Find area of the surface which is the boundary of solid given by $x^2 + y^2 \leq 2x$, $1 \leq z \leq 5$. Draw the picture of this surface.

(3) Compute the flux of the vector field $\vec{v} = (yx, z^3, -yz)$ across the boundary of the solid given by $x^4 + y^4 + z^4 \leq 1$.

(4) Compute the surface integral
$$\int_S (1 + x^2 + y^2)^{1/2} dA$$
where S is surface given by conditions $z^2 = x^2 + y^2 + 1, 1 < x^2 + y^2 < 4$.

(5) Compute the triple integral
$$\int_D |x| dV$$
where D is given by $|x| + |y| + |z| < 1$. Draw the picture.