Practice problems for Midterm 1

Part 1 (chapter 1).

(1) Find the volume of the parallelepiped spanned by the vectors AB, AC, AD where $A = (1, 2, 0), B = (2, 3, 1), C = (-1, 0, 3), D = (0, 2, 0)$.
(2) Write defining equation of the plane that contains the origin and the line given by $x = 3 - 2t, y = 1 + t, z = t$.
(3) Write equation for the line which goes through the origin and is perpendicular to the plane that contains points $A = (1, 2, 0), B = (2, 3, 1), C = (-1, 0, 3)$.
(4) Find the distance from point $A = (1, 2, 1)$ to the plane $x + y - z = 3$.
(5) (*) Write the equation of the line which is bisector of two lines: one contains A and B, the other one contains A and C. We have $A = (1, 1, 2), B = (2, 1, 0), C = (1, 0, 0)$.

Part 2 (chapter 2).

(1) For the curve given by equation $|x|^3 + |y|^3 = 2$, find the tangent line at point $(1, 1)$.
(2) Compute the length of the curve $r = (1 + \cos\phi), \phi \in [0, 2\pi]$, given in polar coordinates. Draw the picture.
(3) For the curve $(\sin t, \cos t, \cosh t)$, find the unit tangent, curvature vector and curvature at point $t = 0$.
(4) The position of a point at time t is given by $(t, \sin t, \cos t)$. The motion started when $t = 0$. At what time the point will travel 1 unit?
(5) Find the length of the curve $(x, x^{3/2}), x \in [0, 1]$.

Part 3 (chapter 3).

(1) The function $f = xy^2$. Find the tangent line to the following level set: $f = 1$ at point $(1, 1)$.
(2) Classify the quadratic forms: $2x^2 + 30y^2 + 5xy, 2xy - 10y^2, x^2 - 2xy + y^2$.
(3) Find the domain and the range of the function $f(x, y) = \sin(\sqrt{xy}) + (x - y)^3$. Explain your answer.

Part 4 (chapter 4, pp. 49–61).

(1) Find equation for the tangent plane for the graph of $f(x, y) = \sin(xy) + x^2 + y^2$ at point $(1, 1)$.
(2) Find the directional derivative of $f(x, y) = e^{x^2+y^4}$ at point $(1, 1)$ in the direction $v = (1, -1)$.
(3) Find partial derivatives of the following functions $f = \log(x^2 + y^6 + 1)$, $f = \arctan(x^2 + y^2)$.
(4) (*) Function $f(r, \phi) = r\cos(\phi^2)$ in polar coordinates. Find $f_y(1, 1)$.